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Life Cycle Activity Analysis: An integrated environmental and economic mathematical programming decision-support model

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Abstract: Life Cycle Activity Analysis (LCAA) - a mathematical programming decision support model for the optimization of the entire life cycle of products - is presented. LCAA is a new tool for the mapping of hierarchical production and recovery chains, their impact on the environment, and for a holistic evaluation of new technologies, environmental strategies or policies. LCAA involves three successive stages of analysis: i) a description of all participating activities (processing, transport, use, recovery, ...) as a good travels from its ‘cradle’ to its ‘grave’, including the inventory of ancillary materials and energy supplied to each activity, economic costs and environmental burdens; ii) the formulation and numerical solution of a linear or nonlinear mathematical programming model and iii) the evaluation of a set of environmental scenarios of interest to policy- decision-makers or stakeholders.

It is shown how LCAA contributes to the conceptualization of Industrial Ecology, which can be seen as a new paradigm for the integration of environmental and economic performance. The antecedents of LCAA (classical Activity Analysis adjoined to the environmental Life Cycle Assessment framework) are surveyed. Illustrative conceptual mathematical programming formats are discussed and the potential of LCAA, the type of problems to be addressed and its relevance to environmental policy are further explored.

Keywords: Activity analysis, environmental policy, industrial ecology, life cycle assessment, optimization.

Introduction

The Industrial Ecology neologism was popularized by Frosch and Gallopoulos in the seminal article Frosch and Gallopoulos (1989). It was proposed that the traditional model of industrial activity in which individual manufacturing processes take in raw materials and generate products to be sold plus waste to be disposed of should be transformed into a more integrated model: an industrial ecosystem. In such a system the consumption of energy and materials is optimized and waste generation is minimized. The industrial ecosystem would function as an analogue of biological ecosystems. According to Frosch and Gallopoulos: “an ideal industrial ecosystem may never be attained in practice, but both manufacturers and consumers must change their habits to approach it more closely if the industrialized world is to maintain its standard of living and the developing nations are to raise theirs to a similar level without adversely affecting the environment.”

In the context of environmental problems, a number of tools for environmental analysis have been developed in the past decades to study the flows of substances, materials and products through the economic system and to assess the associated environmental impacts. Well-known examples of these tools are life cycle assessment (LCA), material flows analysis (MFA), substance flow analysis (SFA), environmental impact assessment (EIA), risk assessment (RA), ... . The purpose of LCA is to study the environmental impacts of a product or a service from the
“cradle” to the “grave” . MFA is used to analyze the materials throughput or the materials intensity of important sectors or large functional systems of the national economy, and therefore concentrates on bulk mass flows. SFA is used to identify the causes of specific environmental problems in the economy and find possibilities for amending or preventing those problems, etc... Many of these tools have different purposes and different systems as their objects, however, in general, they include neither the description of costs nor the mechanisms of economic analysis, Bouman et al. (2000).

On the other hand, the shortcomings of traditional economic analysis have been widely discussed, e.g. Georgescu-Roegen (1973); Ayres (1978); Heijungs (1997); Ayres (1998a, 1998b). In the economic theory of production all quantities are normally expressed in monetary units. This traditional view of the economic process do not explicitly describes the flows and transformation of materials and is unable to deal with unpriced inputs (and outputs), violating the physical laws. It appears that an integration of economic and environmental models is desirable and industrial ecology offers a powerful paradigm for this integration.

The need to integrate physical, environmental and economic models has also been recognized by a number of other authors and this has resulted in the development of various approaches. For example, Leontief (1970); Perrings (1987); Ruth (1993); Heijungs (1997). Other approaches also combine environmental analysis models and optimization methods to identify optimum solutions, namely Azapagic and Clift (1995), Bloemhof-Ruwaard et al. (1996); Leach et al. (1997); Azapagic and Clift (1999).

Jelinski et al. (1992) among others have suggested that Industrial Ecology may be approached in two ways. The first is material-specific, that is, it selects a particular substance, material or group of materials and analyses the ways in which it flows through the industrial systems and, eventually, accumulates in the environmental systems. Examples of this approach are MFA and energy audits. The second type of Industrial Ecology analysis is one that is product-specific. A particular product (process or service) is analyzed in the ways in which its different component materials flows may be modified or redirected in order to optimize the product-environmental interaction. Such an analysis is particularly appropriate at the initial design stage of a product, when decisions on alternative materials or processes are made, or when a product reaches its end-of-life and decisions on alternative waste management strategies must be made. An example of this type of approach is LCA, which is one of the tools used in this research. LCA assesses the environmental aspects and potential impacts of a product, process or service throughout its life cycle, from raw materials acquisition though production, use and disposal, ISO 14040 (1997). This extended system boundary sets LCA apart from other related methods as it provides a full picture of human interactions with the environment. If coupled with economic analysis, LCA can provide a powerful decision-aid tool for an integrated economic and environmental assessment of the material and product supply chains. However, to facilitate this on the practical level it is necessary to develop the appropriate approaches and tools that would help decision-makers and stakeholders to understand: (a) what is the relation between demand of specific products, flows of materials and environmental impacts (b) what changes are needed to reach specific objectives, such us minimizing waste disposal or greenhouse gas emissions and (c) which trade-offs may occur between these disparate aspects in different situations.

This paper is organized in 4 sections, including this introduction. Section 2 describes the antecedents of LCAA - classical Activity Analysis adjoined to the environmental Life Cycle Assessment framework - and presents the main characteristics of the LCAA approach. Section 3 discusses mathematical programming formats. Section 4 summarizes the main findings and contributions of the approach, including the discussion of the limitations and potential further research topics.

### 2. Life Cycle Activity Analysis: Antecedents and Characteristics

Activity Analysis (AA) was developed by Koopmans in the early fiftens, Koopmans (1951, 1957). For this pioneering work, Koopmans received the 1975 Nobel Prize in economics (shared...
with I. Kantorovich). However, the original formulation was not well suited for numerical solution, since it assumed that there were as many commodities as activities, and that the resulting system of equations had a non-singular solution. A major step was the reformulation of AA as a Linear Programming (LP) problem, permitting any number of activities and any number of commodities, Charnes and Cooper (1961).

In an Activity Analysis model, the possible techniques of production available to a firm, or to the economy as a whole, are given by a finite list of elementary activities that can be used simultaneously and at arbitrary non-negative levels. The resulting production possibility set is a polyhedral cone. The activity analysis model, a generalization of the Leontief input/output model, can be used to generate a large number of distinct linear programs, depending on the objective function to be chosen and on the specific set of factor endowments.

Activity Analysis can be viewed as a tool of partial economic analysis modelling for the representation of an industry or a sector of the economy, providing a mathematical format suitable for the representation of an entire vertical production chain, Thore (1991). More recently, Heijungs (1996, 1997) recognized the conceptual similarities between LCA and classical Activity Analysis (AA) and observed that Life Cycle Inventory is an extension of AA, both being "commodity-by-industry analysis", generally seen as superior to other forms of inter-industry analysis, Heijungs (1996), however no connection between mathematical programming and LCA was made. Thus, a major purpose of LCAA discussed here is to highlight how this connection can be established, using extended mathematical programming formats of AA for an integrated economic and environmental analysis of the life cycle of products.

For example, whenever products can be manufactured in alternative ways, distributed through alternative marketing channels, reused or recovered, there exists scope for choice and for controlling the environmental impacts. By combining the LCA approach with mathematical programming techniques, it is possible to represent these options explicitly along the whole supply chain and to solve for optimal economic (e.g. production levels or profit) and environmental performance (e.g. environmental impacts and allocation of resources).

The classical formulation of AA distinguishes three classes of goods: primary goods (natural resources, materials or labor), intermediate goods (outputs which serve as inputs into subsequent activities) and final goods (outputs). LCAA extends the concept of linear activities to embrace mass and energy fluxes over the entire life cycle of products. In particular, the proposed LCAA model includes one additional category: "environmental goods", representing primary resources (material or energy drawn directly from the environment) and emissions of pollutants and the disposal of waste (discarded into the environment without subsequent human transformation).

In the LCA terminology, the “environmental goods” are known as environmental burdens and they can be further aggregated into categories of resource usage and environmental impacts, such as global warming, ozone depletion etc. The purpose of such aggregation is two-fold. Firstly, it interprets the environmental burdens included in the output table in terms of environmental problems or hazards. Secondly, by aggregating a large set of data into a smaller number of impact categories it simplifies the decision-making process.

LCA originates from energy analysis studies, first published in the 1970’s, which considered only energy consumption over the life cycle of a product or a process, e.g. Boustead (1972) and Boustead and Hancock (1979). Over the last decade LCA has received wider attention and intensive methodological development leading to the publication of several documents and guides. Examples include an LCA guide published by the CML, Heijungs et al. (1992), and the SETAC “Code of Practice”, Consoli et al. (1993). More recently, the International Organisation for Standardisation (ISO) has adopted environmental management standards for LCA, ISO 14040 (1997); ISO 14041 (1998); ISO 14042 (1999); ISO 14043 (2000).

In a recent guide, updating the previous guidelines, Guinée et al. (2001), LCA is defined as “a tool for the analysis of the environmental burden of a product at all stages in their life cycle – from the extraction of resources, through the production of materials, products parts and the product itself and the use of the product to the management after it is discarded, either by reuse, recycling or final disposal.” The LCA methodology has four components: (1) goal defini-
tion and scoping, (2) inventory analysis (also, life cycle inventory), (3) impact assessment and (4) interpretation. A full life cycle assessment includes each of these four components. Detailed information about the LCA methodology can be found, for example, in the previously mentioned references and thus the text below focus on some of the concepts with more relevance for the development of the LCAA methodology. For example, the concepts of “foreground” and “background” proposed within the environmental systems analysis theory are very useful since they help to distinguish between unit processes of direct interest in the study, and other operations with which they exchange materials and energy, Clift et al. (1999, 2000). The foreground may be defined as the endogenous part of the production chain, which includes the set of processes whose selection or mode of operation is affected directly by the decisions of the study. The background denotes the exogenous parts of the production chain, comprising all other processes that interact directly with the foreground system, usually by supplying material or energy to the foreground or receiving material and energy from it. These concepts are illustrated in Figure 1.

Adopting these concepts and terminology, a complete life cycle approach must pursue the production chains both upstream (all the way to their "cradle") and downstream (to their "grave"), by explicitly encompassing the indirect effects associated with the supply of goods together with direct effects of the core system being modeled. Section 3.2 describes how the LCAA approach is formulated to account not only for the environmental burdens of processes in the foreground but also for impacts in the background. Thus, the total environmental impacts are calculated over both the endogenous and the exogenous part of the life cycle. The foreground and background concepts are also useful in setting goals and targets which can be attached to both variables in the foreground and in the background.

Figure 1. Foreground and background systems

LCAA is a new tool for the mapping of hierarchical production and recovery chains, their impact on the environment, and for a holistic evaluation of new technologies, environmental strategies or policies. LCAA involves three successive stages of analysis: i) a description of all participating activities (processing, transport, use, recovery, ...) as a good travels from its "cradle" to its "grave", including the inventory of ancillary materials and energy supplied to each activity, economic costs and environmental burdens; ii) the formulation and numerical solution of a linear or nonlinear mathematical programming model and iii) the evaluation of a set of environmental scenarios of interest to decision-makers or stake-holders.

Furthermore, LCAA considers alternative methods of production, distribution, reuse and recovery. It provides a general optimization framework for the modelling of reuse and recovery of products, leading to loops in the life-cycle chain. It permits materials and energy to be recovered both inside the foreground (closed-loop) and between the foreground and the background.
open-loop). In addition, the recovery of materials or energy in the foreground may displace activities in the background system, leading to the avoidance of environmental burdens that would otherwise have been generated. This is known as the “avoided burden” approach [e.g. Clift and Doig (1996)] and that can also be expressed explicitly within LCAA.

The LCAA approach offers the following advantages:

- LCAA explicitly recognizes the possibility of alternative ways of production (e.g. using raw or recycled materials), alternative distribution channels, and alternative reuse or recovery processes. In other words, there are mathematical degrees of freedom present in the model format, and the purpose of the programming model is to determine the optimal choices at each stage of the logistics chain.

- LCAA incorporates advanced techniques for representing environmental goals in the model, in the so-called Goal Programming. These goals do not need to be absolute targets but can be a set of ordinal priorities, laid down by decision- or policy-makers. Goals can also be ordered in hierarchies, with sub-goals in several layers. In this manner, LCAA allows for realistic modelling of the goal-based process.

- Through the LCA approach, individual environmental burdens are brought together under environmental impact categories, such as global warming, acidification, etc. Using LCA terminology, this is equivalent to Life Cycle Impact Assessment (LCIA), ISO 14040 (1997). However, unlike in LCA, there are mathematical degrees of freedom present in the LCAA model, because vectors of individual environmental burdens associated with alternative activities may translate into the same aggregated impacts. The optimizing format makes it therefore possible to determine the optimal levels of individual environmental burdens (and associated vector of activity levels) that are commensurate with a given set of goals in terms of environmental impact categories.

- LCAA is a numerical technique, requiring the use of advanced mathematical programming software. The software used in this work is GAMS (General Algebraic Modelling Software), Brooke et al. (1998).

Varying the numerical assumptions of the model (and varying the goals or the priorities parametrically), LCAA can be used to generate a set of scenarios to be presented to the policymaker. In this manner, a series of “what if?” questions can be addressed and answered.

The conceptual foundations for LCAA are evident and have been described in the beginning of this section. However, it should be noted that the research methodology followed has mainly been “applications-driven”, meaning that relevance was attained by starting with concrete problems in the context of actual applications (cf. Cooper and McAllister (1999)). The validation and generalization of the methodology has been made through its application to different case studies. The application of LCAA to bottled water, scrap tires and plastic panels mounted on electronic equipment, including numerical results and discussion, can be found in Freire et al. (2000, 2001 and 2002), respectively. The analysis of mathematical programming formats that can be formulated within the LCCA framework is presented in the next section.

3. Mathematical Programming Formats

Depending on the type of applications and problems to be addressed, different types of models can be formulated. For example, many alternative objective functions can be specified (or even a multi-objective approach) using linear and non-linear programming techniques. Two simplified versions are presented as illustrative examples of the type of programming models that can be formulated. The first version considers only closed loops while the second one includes the possibility of open-loops (recovery from the foreground to the background).

The LCAA model uses an input-output format. A detailed notation list can be found at the end of the paper. A basic mathematical format of LCAA can be written as the following linear program:

\[
\min cx + qw
\]
subject to 

\[-A^P x + w \geq 0\]

\[-(A^I + B^I) x = 0\]

\[B^F x d \geq d\]

\[(A^E - B^E) x - Dw \geq -g\]

\[x, w \geq 0\]

(1)

where (see also Notation in the appendix) \(cx\) represents the total costs of operating the activities \(x\) and \(qw\) is the total cost of primary goods. \(A\) and \(B\) are matrices of input and output coefficients, respectively; \(w\) represents a column vector of supply levels of primary goods, such as material and energy from the background system. Superscripts \(P, I, F\) and \(E\) represent primary, intermediate, final and “environmental goods”, respectively. Primary goods are inputs of products, material and energy produced in the background. Intermediate goods are outputs that serve as inputs into subsequent activities, either in the foreground or in the background. Final goods are the functional outputs delivered by the distributed and purchased products, the production of which is the objective of the economic system under study. “Environmental goods” or interventions are flows of materials or energy drawn from or discarded into the environment without subsequent human transformation. By convention, the input coefficients \((A\text{-coefficients})\) have a minus sign and the output coefficients \((B\text{-coefficients})\) are assigned a positive sign. Consequently, matrices \(A\) and \(B\) become partitioned into:

\[A = (\begin{array}{c}
-A^P, -A^I, 0, -A^E
\end{array})\]

\[B = (0, B^I, B^F, B^E)\]

(2)

As discussed in the previous section, the model adopts the concept of the foreground and background systems (see Figure 1). The foreground is modeled in some explicit detail: the production activities themselves, and the conversion of intermediate goods into final goods, i.e. the set of processes whose selection or level of operation can be affected directly by decisions in the study. The background comprises the exogenous flows of the model, i.e. the supplies of primary goods.

The “environmental goods” or interventions arising from the foreground (i.e. from the operations which are being modeled directly) are termed direct burdens. They include the direct emissions from operating the activities (e.g. combustion, chemical reactions, thermal treatments, long-term leachate emissions from landfill etc.) and from the transportation of intermediate goods. The resource usage and emissions arising from the background activities are termed indirect burdens; they are caused by the changes in the demand of products, materials and energy in the foreground. The indirect burdens can be described by generic industry data, obtainable from commercial or public life cycle inventory databases. Direct burdens on the other hand are process-specific and must be sourced from the manufacturers in the foreground.

In this way, the model calculates the total accumulated environmental burdens over the entire life cycle of the product, including the indirect environmental burdens of primary goods arising in the background. Thus, the total environmental burdens arising over the life cycle of the products are equal to the sum of the foreground (direct) burdens and the background (indirect) burdens, that is \((-A^E + B^E) x + Dw\), where \(Dw\) is a vector of environmental effects arising from the background.

The model (1) minimizes total costs, which comprise the costs of operating activities and of primary goods. For present purposes, it is assumed that the prices of all primary goods are known and constant.

The crucial feature of formulation (1) is the constraint \((A^E - B^E) x - Dw \geq -g\), which requires the environmental burdens \((A^E - B^E) x - Dw\) not to exceed a vector of environmental goals \(g\) set for example by a policy- or decision-maker.

The second version of an LCAA mathematic programming format involves expanding the possibilities for reuse and/or recovery of products. As mentioned before, such loops in the life cycle chain can take two forms: recovery entirely in the foreground (closed loop) and recovery from
the foreground to the background (open loop). Materials and energy recovered in the foreground, which are also inputs to the activities in the foreground (closed loops), may lead to the avoidance of environmental burdens. This is the case when burdens associated with foreground activities that are displaced by the recovery processes are higher than the burdens of the recovery itself. The opposite is also possible: material loops may sometimes lead to higher environmental burdens, i.e. a worse environmental performance overall. This can happen when the recovery of used products and materials by itself imposes considerable burdens.

A product that is recovered and exchanged with the background system will be treated as an intermediate good. The usual assumption is that the recovery of materials and/or energy in the foreground does not affect the demand for goods and services in the background (except for materials and energy supplied to the foreground activities), Clift et al. (2000). Therefore, the market balance for intermediate goods which was defined in (1) as \((- A^i + B^i)x = 0\) has to be amended to \((- A^i + B^i)x - y = 0\), where \(y\) is a column vector of unknown levels of recovery of intermediate goods. Zero entries indicate recovery entirely in the foreground, positive entries indicate recovery supplied from the foreground to background.

Adopting these assumptions, the total environmental burdens are then equal to the sum of the foreground (direct) burdens and the background (indirect) burdens minus the avoided burdens, that is: \((B^e - A^e) x + Dw - Dy\).

Regarding economic considerations, when recovery or reuse occurs entirely in the foreground, no additional net revenues or costs accrue, since these economic flows have already been taken into account in the activity analysis format. However, when intermediate goods are recovered back to the background and thus “exported” to the exogenous part of the model, it is necessary to account for the net revenue (or net cost) \(p y\), collected in the foreground, where \(p\) is a vector of unit prices of recovered goods. Here, \(p\) is assumed to be known and to represent average prices of recovered goods. (Alternatively, marginal prices or price sensitive functions could be used, describing the price elasticity of recovered goods. The latter extension would cause the model to change from a linear to non-linear one.) Combining these changes to accommodate recovery of goods, the programming format (2) becomes:

\[
\begin{align*}
\text{min} & \quad cx + \bar{q}w - py \\
\text{subject to} & \quad \begin{aligned}
- A^i x + w & \geq 0 \\
( - A^i + B^i)x - y & = 0 \\
B^e x & \geq d \\
(A^e - B^e) x - Dw + Dy & \geq -g
\end{aligned} \\
& \quad x, y, w \geq 0
\end{align*}
\]

Programming format (3) represents the extended LCAA format, accounting for the possibility of closed-loops. Further extensions to these two basic model are possible. For example, transportation and shipping of goods between various locations may be accounted for in all parts of the supply chain. The basic programming format still applies, treating each transportation link as a separate activity, with its own inputs and outputs, Freire et al. (2001). Moreover, if the time-profile of activities is important, the model may be developed into a multi-period one. All variables then need to be dated, and the market balances in each time period need to be defined explicitly.

**Environmental Life Cycle Impact Assessment**

The \(B^e\) and \(-A^e\) matrices constitute an inventory table, summing up the outflows and subtracting the inflows of “environmental goods” associated with the economic activities. In LCA, this is part of Inventory Analysis.

Flows of substances are recognized as environmental problems only when they pose problems to the environment and society. Thus, there is an intrinsic value-bound aspect to the definition of an environmental problem, Heijungs (1997). To deal with this, it is necessary to establish sci-
Scientific relationships between pollutants and a set of environmental impact categories, such as the greenhouse effect, acidification or ozone layer depletion. Similarly, there is a relationship between resource extraction and various depletion problems. Hence, the impact categories can be defined in terms of damage to the environment by pollutants in air, water or soil and by the depletion of available natural resources. In LCA terminology, aggregation of environmental burdens into impact categories is carried out in the Impact Assessment phase.

As described by the environmental-goal constraint in the extended program (3), the vector of environmental burdens, \( E(i) \), is equal to the sum of all direct and indirect burdens minus the avoided burdens:

\[
E(i) = (B^E - A^E)x + Dw - Dy
\]

where \( i \) represents individual environmental burdens. The individual burdens can be aggregated into a set of environmental impact categories according to the expression:

\[
I(j) = F(j,i) . E(i)
\]

where \( I(j) \) is a vector of environmental impact categories \( j \) and \( F(j,i) \) is a matrix of relative impact coefficients (for example, the global warming impact coefficients of greenhouse gases are expressed relative to \( CO_2 \), whose coefficient is defined as unity).

The environmental goal-oriented expression may then be reformulated into:

\[
F(j,i) . [(A^E - B^E)x - Dw + Dy] \geq -g'
\]

where \( g' \) is a vector of goals defined directly in terms of environmental impact categories:

\[
g' = F(j,i) . g
\]

More advanced formulations are also possible. For example, as proposed in Freire et al. (2001), by treating the vector of individual environmental goals \( g \) as an unknown rather than as a given parameter. This means searching out an optimal combination of individual goals – i.e. an optimal combination of individual environmental burdens – possibly trading-off one individual burden against another while still satisfying the goals defined for the impact.

4. Conclusions

The present work has demonstrated the potential of a novel tool – Life Cycle Activity Analysis – for an integrated economic and environmental analysis of the material-product chains associated with the life cycle of products. This tool combines the advantages of the Life Cycle Assessment (LCA) methodology, that tracks the environmental consequences of a product, process or service from “cradle” (resource origin) to “grave” (final disposal), with the advantages of using mathematical programming formats of economic Activity Analysis. The methodology allows for the analysis of “What if?” scenarios. In this manner, it can be used to design and evaluate alternative packages of environmental strategy or policy, including programs of action for recycling and reuse of products, with the aim of identifying more sustainable practices for the future.

To explain the relationship between LCAA and conventional LCA, it can be pointed out that Life Cycle Assessment is a special (mathematically: a degenerate) case of LCAA. Both techniques track the environmental impacts of a product from the original use of natural resources to the final disposal of the used product. LCAA extends the LCA framework by recognizing the possible presence of alternative activities along the cradle-to-grave logistics chain and by including economic costs. There are then mathematical degrees of freedom present in the configuration of the chain, and choices have to be made. LCAA represents this choice situation as a task of optimization of a mathematical programming model. Points along the product chain where choices are to be made and alternative activities can be chosen are forking points in the product flow. There exist in principle two kinds of such branching situations: (i) simple (linear) alternative routings and (ii) the possibility of reuse of a product; recycling of its component materials or recovery of its energy content (introducing loops in the product flow). However, just because reuse or recovery is technically feasible, it does not follow that they are environmentally or economically
superior. LCAA will evaluate all possible reuse and recovery opportunities available and will select those routings of the product flow that minimize total costs and commensurate with the environmental targets defined. Additionally, this joint format allows for the quantification in financial terms of the costs associated with limitations imposed (through determination of the respective shadow costs).

Further Research and Limitations of the Approach

The mathematical models presented assume linearity, which is acceptable when the life cycle of the product being modeled induces only marginal changes in the activity levels of the operations being considered. However, if specific aspects of the life cycle are acknowledged as non-linear, then non-linearity has to be introduced and defined mathematically in an appropriate manner.

The data chosen for the numerical illustrations in the present paper were supposed to be time-independent. This assumption is also used by standard approaches such as LCA and MFA. It may be permitted when short (up to one year) product life cycles (including disposal and recovery) are considered. However, many life cycle problems involve much longer time spans, simply because many products are durable and last for decades before they are disposed of. Further complications are introduced when processes and products gradually change over the long run. It may be possible to deal with such situations by estimating the LCAA model using time series data for time-dependent variables. Unfortunately, lack of time series data may strongly limit the extension of LCAA to include such dynamic issues. For both static and dynamic models, accuracy and completeness of data is a very important. In the absence of reliable data, both the LCAA analysis and the assessment of its results will be seriously hampered.

The considerable amount of information needed by the LCAA model requires the co-operation of many different specialists. The industrial engineer's approach operating on process or plant level and focusing on logistics and cost accounting will be one ingredient in this joint effort. The economist's approach operating on regional or macro economic level will be another. The environmental scientist/engineer evaluating environmental impacts needs certainly to be integrated. All these contributions need to be brought together in a complementary fashion.

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Notation

A  matrix of input coefficients; each element denotes the quantity of an input required to operate an activity at unit level

B  matrix of output coefficients; each element is the quantity of an output obtained when an activity is operated at unit level

c  row vector of unit costs of operating the various activities, it is known and given

d  column vector of final demand, it is known and given;

D  matrix of unit environmental burdens; each element is the environmental burden generated in the upstream processing, transportation and manufacture of one unit of primary goods

F(j,i) matrix of relative environmental impact coefficients

g  a vector of environmental goals defined in terms of burdens

g’  a vector of goals defined directly in terms of environmental impact categories, g’ = F(j,i).g

p  a row vector of unit prices of recovered goods
\( q \) a row vector of unit costs of primary goods

\( w \) a column vector of supply levels of primary goods, such as material and energy from the background system

\( x \) a column vector of unknown activity levels

\( y \) a column vector of unknown levels of recovery of intermediate goods; zero entries indicate recovery entirely in the foreground, positive entries indicate recovery supplied from the foreground to background

**Superscripts**

E “environmental goods”

F final goods

I intermediate goods

P primary goods

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