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Jayet P.-A.

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Mathematical programming for the computation of multi-market equilibria: An attempt in modelling the French markets of cereals and livestock feeds

Pierre-Alain Jayet
UMR Economie Publique, INRA

Abstract: Sectorial models based on linear programming are frequently developed for the agricultural production and animal feed industry sectors. The analysis proposes the interaction of two models to represent each of these sectors, given the fact that one supplies what the other demands. This multi-market modelling is used to determine equilibrium prices and quantities. Emphasis is placed on the operational character of the process while several of the limitations inherent in the approach are pointed out.

Keywords: Linear programming, agriculture, multi-market modelling, livestock feeds, France.

Introduction

The transformation of the Common Agricultural Policy (CAP) in the wake of the reform agreements of the 1991-2001 decade, along with the GATT shift towards the World Trade Organization (WTO), has modifed the classical reference marks of the agricultural raw materials market. Indeed in markets that are henceforth much more affected at the global level, large price fluctuations can disrupt the function of food industry transformers. The gap is growing between the perfectly structured agricultural markets with stable and guaranteed prices on the one hand, and the food industry markets downstream that are subject to the fluctuations of global markets on the other.

Developments in the cereal and livestock feed markets mainly and directly concern two types of economic agents farmers livestock producers and live stock feed producers. Two repercussions are inevitable for each of these types of agent and for each of the product categories, since the products are both production factors and products of transformation. Livestock feed producers are highly dependent on the price levels of cereals and on substitution products. In fact raw materials account for 70-80% of livestock feed costs. The prices of feed and cereals contributing to such costs thus weigh heavily on farmers and livestock producers' margins, as well as on those of the feed producers.

Livestock feed is one of the main outlets for cereals, and livestock raising is obviously the main outlet for the livestock feed production sector. Now livestock raising and farming are closely intertwined, either through livestock feed of industrial origin or through the integrated on farm consumption of cereals

produced on the farm. The cereal and industrial feed markets can, thus, be well represented by a model that takes into account the interactions of the raisers, the farmers and the feed producers. The MODANI model (Birfet, Jayet and Lapierre, 1999) was developed with the goal of determining the simultaneous equilibrium of these products in their markets by relying on two specific models one from the farming sector and the other from the industrial livestock feed sector. The first (AROPAj; see, for example, De Cara and Jayet, 2000) is a supply model for farming and raising products. The second (Prospective Aliment) is initially a demand model for raw materials, among which cereals are preponderant.

The two types of actors and two sectors took and still take advantage of models based on mathematical programming, specifically that of linear programming. To exemplify this long experience, a few early works can be cited (Shechter and Heady, 1970), then those concerned with high scale modelling of the agricultural sector (Hertel, Preckel and Huang,1989), along with works on the related problems of aggregation (Day, 1969); Paris and Rausser, 1973 ; Spreen and Takayama,1980). Agriculture and the environment still take advantage of analyses based on models of this type (De Cara and Jayet, 2000). As far as livestock feed is concerned, the literature also contains works on the subject, but they are treated in a somewhat more confidential manner (Kearney, 1971).

We here propose a synthetic approach to the modelling principles underlying the MODANI model by completing it through the presentation of practical modalities in order to calculate simultaneous balance on a large number of markets. Section 1 presents the two models which we intend to couple and also discusses how to overcome the pairing problems inherent in this procedure. The coupling modalities are explained in section 2. Section 3 is dedicated to the presentation of results.

Sectorial supply and demand models

The AROPAj model represents the sectorial supply in agriculture and livestock raising. In reality this model integrates the sales of the main annual crops, that is to say, cereals, oilseed crops, sugar-beets and potatoes, along with pastureland and fodder and, finally, the main livestock productions (cattle, sheep and caprines, hogs, poultry). It also integrates the daily consumptions of the livestock, whether it be of raw products from the farm (cereals) or purchased concentrates or basic feeds. In the model, the concentrated feeds comprise four different types of compound and simple feeds, depending on the livestock for which they are intended.

A multiple representation is given of the sector in the sense that it combines a group of typical producers dierentiated according to the region and the technico-economic orientations defined in the nomenclature of the Farm Accounting Data Network (known with its french acronym RICA: *Réseau d'Information Comptable Agricole*). For each typical producer a linear program is assigned to maximise gross margin under constraints grouped into four modules: (i) an 'agronomic' module (crop rotation), (ii) a livestock module, (iii) a 'demographic' module (only for the cattle herd), (iv) a module representing the economic constraints associated with the Common Agricultural Policy (CAP).

A typical producer is characterized by a range of parameters, the values of which are given by an expert or are obtained according to estimates based on RICA data. The CAP is integrated into most existing integration modalities or foreseen for future reforms (quotas land limitations 'surface' compensation, direct aids, co-responsibility taxation, extensification incentives) through a range of parameters common to all of the typical groups. The integration of agricultural policy constraints necessitates the use of binary and integer variables. At the level of numerical techniques, switching from parameter values on the one hand to matrix coefficients, objective functions and 'right hand side' values on the other is managed by a program and matrix generator. Once the matrices have been designed according to normal standards the 'solver' makes it possible to optimize the dierent linear programs.

From the economic analysis point of view and for our interests here AROPAj makes it possible to elaborate the cereal supply functions and the livestock feed demand functions. By varying prices it is, in fact, possible to reconstitute these functions with the classical linear programming particularity of obtaining 'staircase' functions[1] .

[1] In reality, on the basis of this type of linear program, it is possible to build supply or demand functions that are no longer 'stage' functions even if they remain discontinuous. The elaboration module for this type of function is not yet associated with the operational version of the model. It will involve the inclusion of yields as functions of inputs, such as nitrogen fertilizers, and the computation of the optimal level of inputs induced by prices of inputs and prices of outputs.

The pertinent 'AROPAj' activities for modelling the markets are mainly cereal gathering (soft wheat, barley, grain corn), the consumption of simple or compound concentrated feeds (four types) and the consumption of basic feed (only one type). Secondly, other activities are auto-consumption of cereals (associated with the three previous cereals), the on-farm consumption of fodder and livestock activities, as well as productions associated with by-products of interest to the livestock feed sector (for example, beetroot pulp, cereal straw).

On the industry side of the markets, the ALIMAj model makes it possible to obtain the feed supply and raw material demand functions. This model is a transformed version of the Prospective Aliment model (a presentation and application of this model can be found in Lapierre and Pressenda (2000). Initially, it is a case of optimizing the purchase of raw materials so as to satisfy the needs of a given feed production, while respecting certain standards in terms of energy and protein content. It therefore represents the classical problem of minimizing costs.

More precisely, the constraints are grouped into three modules: one concerning feed characteristics, one the limits of incorporation and the last one regional demands satisfaction. On the basis of the same production set the ALIMAj model is a version of the Prospective Aliment model transformed mainly at the objective function level and by the addition of a capacity constraint. The objective function becomes the dierence between the sales product of the transformed feeds and the purchase costs of the raw materials. The capacity of an industrial system is, in fact, defined by the global quantity of feed produced. This capacity is, in a way, the counterpart of 'land' or 'livestock' capital that characterizes the AROPAj model.

The ALIMAj model integrates 23 feed compounds and 37 raw materials susceptible of entering into the industrial composition of these compounds. The cereals for which livestock feed is the main outlet, are obviously found among the raw materials, hence one could elaborate cereal demand functions for the livestock feed. Symmetrically, one could dispose of the transformed livestock feed supply functions.

Similar to AROPAj, this model 'aggregates' a set of autonomous models, each of which is representative of an industrial region. At the data processing techniques level, the same principles for generating programs and matrixes are implemented as those developed for AROPAj.

Coupling modalities

The following briefly describes how the MODANI model couples with the AROPAj and ALIMAj models (for additional information see Chapter 3 in Birfet et al. (1999) or Chapter 4 in Jayet and Lapierre (1998). This coupling is carried out using a version of the AROPAj model with 19 agricultural regions (Figure 1) subdivided into 82 typical groups of producers ; the ALIMAj model used is based on 9 industrial regions in which 22 administrative regions can be found. The ALIMAj regions are Bretagne, the Loire River - Poitou Charentes region, Normandie (Upper and Lower Normandie), the 'Southwest' (Midi-Pyrenees and Aquitaine), the 'North' (North-Pas de Calais, Picardie, Ile-de-France, Champagne-Ardennes), the 'Large East' (Lorraine, Alsace, Franche-Comté), the Center – Burgundie, the 'Southeast' (Rhones-Alpes, Provence-Alpes, Riviera, Languedoc-Roussillon, Corsica) and Auvergne-Limousin.

The non-superposition of the two regional divisions poses the initial problem of pairing and compatibility. Each industrial region was assumed to be in a position to sell concentrated feed to each of the farming agricultural regions, the prices being differentiated by transport costs calculated for this purpose.

The second pairing problem concerns the products 'exchanged' by the two models. Livestock activities (especially hogs and poultry) and feed purchased for livestock are defined in a relatively aggregated manner by the AROPAj model, whereas the level of feed characterization is very precise in the ALIMAj model. The feed markets as they are represented here are, therefore, markets of aggregated products.

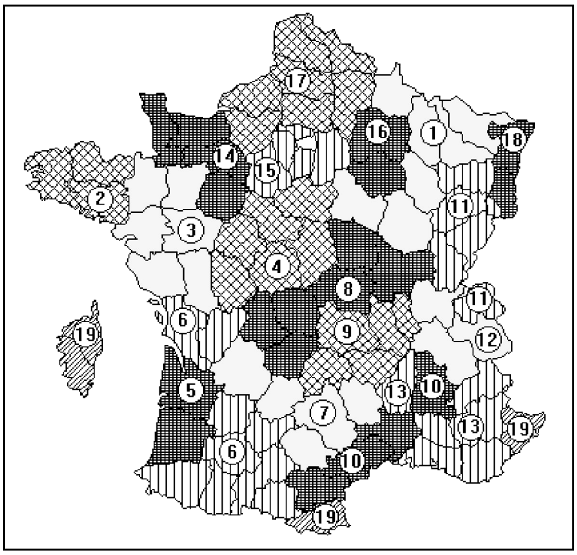

Figure 1. Regions of the AROPAj model-grid in the AROPA2 version.

Nine industrial feeds are selected : two types of simple feeds (simple 'energy' feeds and simple 'protein' feeds) and one or two types of compound feeds for each of the 5 large categories of livestock: hogs ('energy' and 'protein' feeds), poultry (a compound feed), 'milk' cattle ('energy' and 'protein' feeds), 'beef'cattle (a compound feed), sheep and caprines (a compound feed).

The raw materials for the livestock feed industry also pose pairing diffculties. The AROPAj model focuses on the main products for which harvesting is regulated by the CAP. In general, the harvest and transformation of these products is accompanied by by-products that are not necessarily concerned by market exchanges involving the agricultural producers (gathering cereal straw does not represent a significant level of commercial flow since such straw is often re-used on the farm; as for the principal by-products they appear at the level of cooperatives and transformation industries). Such by-products represent important raw materials for the livestock feed industry.

The 37 raw materials emplyed in the ALIMAj model are grouped into five categories according to their agricultural or non-agricultural origin. We distinguish between the agricultural raw materials that can be incorporated directly into feeds and that are of European origin (cereals, cereal straw, peas, sorghum, rape), agricultural raw materials from outside Europe (manioc, soja), raw materials from the food-industry produced either in France or imported and raw materials of non-agricultural origin. In terms of supply for industry elements of the first category can be considered as being represented by AROPAj. The elements of the third category will be evaluated on the basis of technical transformation coefficients applied to the quantities collected at the exit of agricultural exploitation. The other categories refer to simple importation models that take into account the concurrent domestic uses (industries producing foodstuffs for humans, other industrial sectors). Product by product, all importing, exporting or use other than in the livestock feed industry is of such a quantity as to give an elasticity price close to the value 1 at the level of the initial price and tending towards zero whenever the prices distance themselves.

Once the products have been characterized according to origin and destination, and in terms of price, each of the AROPAj and ALIMAj models is capable of determining the supply and demand of the products in question. Nevertheless, taking into account that the AROPAj and ALIMAj models were separately calibrated at the levels of groups or typical regions, the initial prices generally do not correspond to equilibrium prices. The sets of aggregated production compatible with the set of constraints, particularly the capacity constraints, do not make it possible to reach competitive equilibrium in the different national markets (Figure 2 shows the dierent possible configurations affecting two raw materials produced by the agricultural sector and consumed by industry. In some markets, adjustment by international exchange or by transfer to different sectors appears unsatisfactory compared to what happens in reality. Moreover, calculating equilibrium is the goal sought by all types of scenarii proposed in terms of technical progress or when changes are made to agricultural policies.

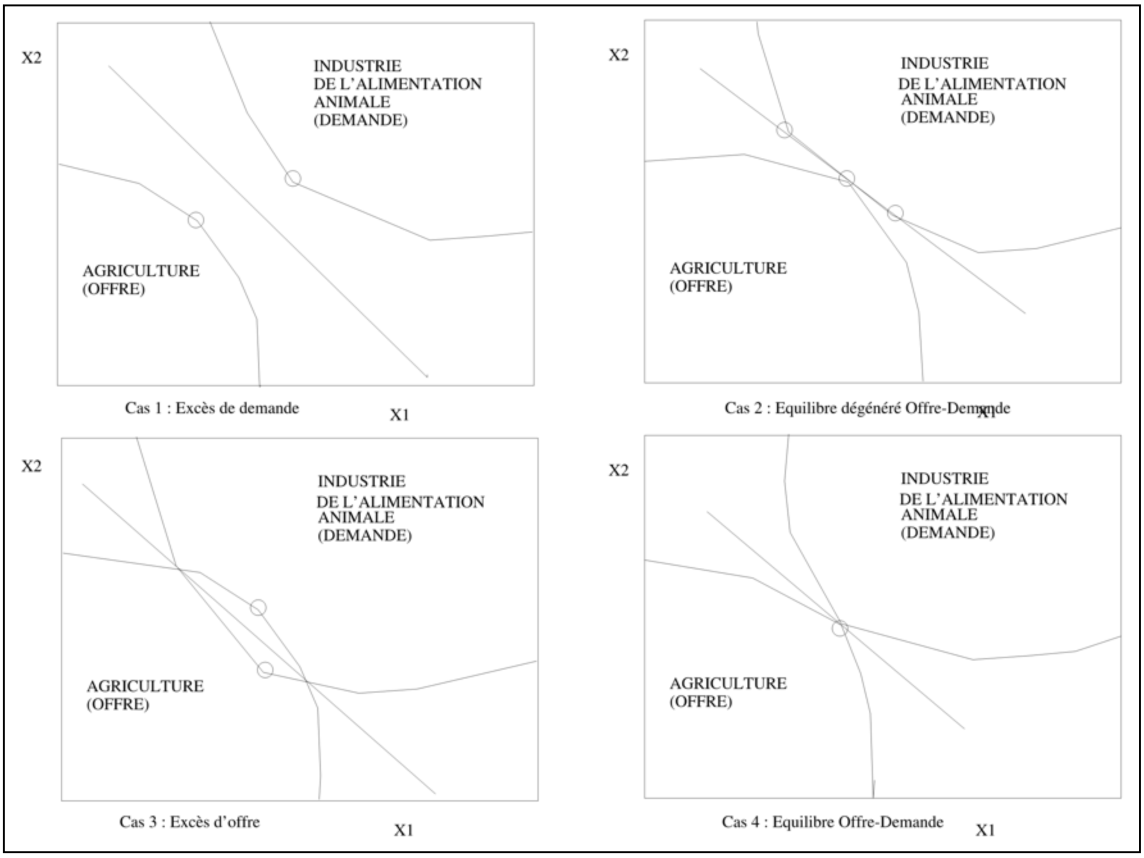

Figure 2. Different cases in balance in two raw materials markets according to the price vector and the isoprofit curves on the two sides of the markets.

It is therefore necessary to propose a method which will make it possible to calculate or to approach equilibrium prices and quantities in a large number of markets. To this effect, we retained a procedure of iterative tentatives, making it possible to approach equilibrium on the basis of hypotheses concerning excess demand price elasticities and adjustments in capacity. In principle, excess demand that appears in a market during one iteration of the procedure will result in a price variation of the same sign for the corresponding product in the next iteration.

Along with price changes, the capacities are adjusted. More precisely, considering that the industrial sector is more 'reactive' than the agricultural sector, positive excess demand in the concentrated feed markets during one iteration will lead to an increase in industrial capacity in the next iteration. An excess in negative demand on these markets will lead to a decrease in industrial capacity. The repercussion on the regional industrial capacities will be proportional to the initial regional capacities. As far as the livestock raisers are concerned, only livestock capital is adjustable with a predetermined maximum amplitude. Farming surfaces are not modified.

At each iteration, calculation of the supplies and demands is thus made by each model (i.e. typical farms on the AROPAj side and regions on the ALIMAj side) on the basis of the prices and readjusted capacities with all the actors regarded as 'price takers'. The MODANI model is, in fact, a model that combines the call for the AROPAj and ALIMAj models, calculates the aggregated supplies and demands (then the demand excesses) and adjusts the prices and capacities. Once the initial capacities and prices have been defined, the iterative calculation of the national demand excesses in the different markets results in a trajectory obtained according to a determinist procedure of tentatives which is itself empirically defined. A short presentation of the two basic models is given in the Appendix.

A few results and criticisms

A few simulation results obtained in very different agricultural policy contexts are given so as to show the operational character of the MODANI model. Empirically, a few elements are added to the running of the simulations as adjustments. They concern mainly the industrial capacities for which the gap between minimum and maximum capacities cannot exceed 80% of the initial capacities. As far as the change in price in each of the markets is concerned, it is fixed at 25%. During the first iteration, it is divided by 2 as soon as the excess demand changes sign. It is a question of limiting the impact of jumps and stages inherent in linear programming. We should also take account of the fact that the AROPAj and ALIMAj models have been calibrated according to dierent data bases. Particularly AROPAj, based on the RICA, must be given a scaling coefficient. In fact, the RICA is not sufficiently representative of certain segments of supply (for example, part-time producers).

One of the most difficult practical problems to solve is to select the price values of certain products that are compatible with both the estimations furnished by AROPAj and the transport costs specifically defined for the MODANI model. This point perhaps furnishes an a priori explanation of the large differences between the initial and the equilibrium price observed in certain markets when the iterative procedure makes it possible to approach it. As far as the raw materials are concerned, it was decided to fluctuate the prices (up) even though the latter are strongly structured by the CAP. It is then a question of respecting the regulations which, in this case, operate like a price threshold and of showing to what extent the model is capable of integrating agricultural price fluctuations.

Three agricultural policy scenarios were explored, namely (i) a scenario based on the CAP in effect in 1990 (as the version of the AROPAj model used in MODANI was calibrated using RICA data), (ii) a scenario based on the 1995 reformed CAP, (iii) an 'Agenda 2000' type scenario based on reform projects as discussed in 1998.

The number of iterations is fixed at 60 for each of the scenarii, but the stabilization of the procedure improves only slightly after 40 iterations. To illustrate this point, Figures 3 and 4 give the trajectories drawn up at the quantity level, and the price level, which associates both price and excess demand for several products with the trajectories limited to the first 40 iterations. Products were chosen according to their importance in the exchanges, or depending on the type of trajectory projected by the simulations.

Equilibrium is approached whenever excess demand approaches zero simultaneously in all markets. Such was the case for the quasi-total of the 37 raw materials in the 3 scenarios. It was also the case for 6 out of 9 of the livestock feeds (for 7 of them in the 'Agenda 2000' scenario). One notices that the feeds posing a problem in seeking equilibrium were those associated with livestock productions and with the typical Groups, the worst being represented by AROPAj (poultry, sheep and caprines).

The oscillations observed in certain markets clearly relate to the specificities of linear programming and to the resulting supply and demand functions. One can also suppose that the choices made in the modalities governing the tentatives strongly affect the trajectories. Assuming that the modeler is an expert in the markets examined here could hence, through his choices, anticipate the equilibrium obtained. Over and above the calibration of the AROPAj and ALIMAj sectorial models, it would thus be possible to calibrate the MODANI model. However, it is important to ensure that all the required price and quantity data are available in order to characterize the exchanges in all of the markets.

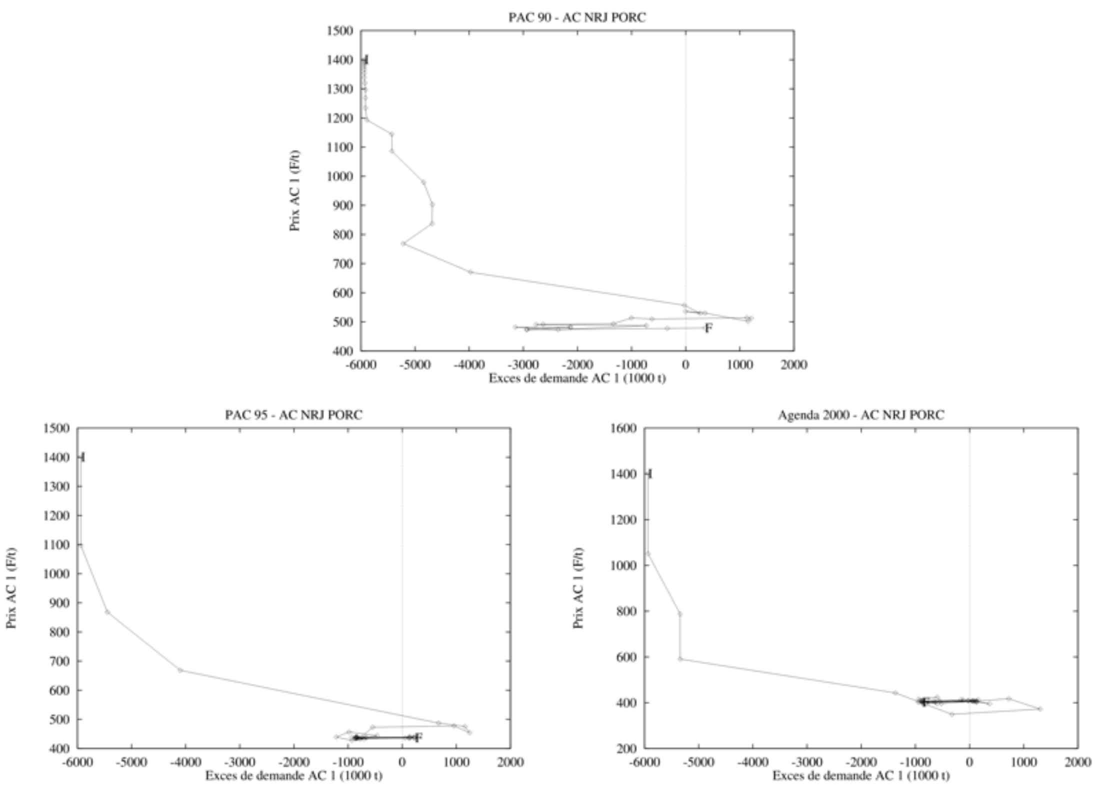

Figure 3. The market of feeds “energy for hogs”

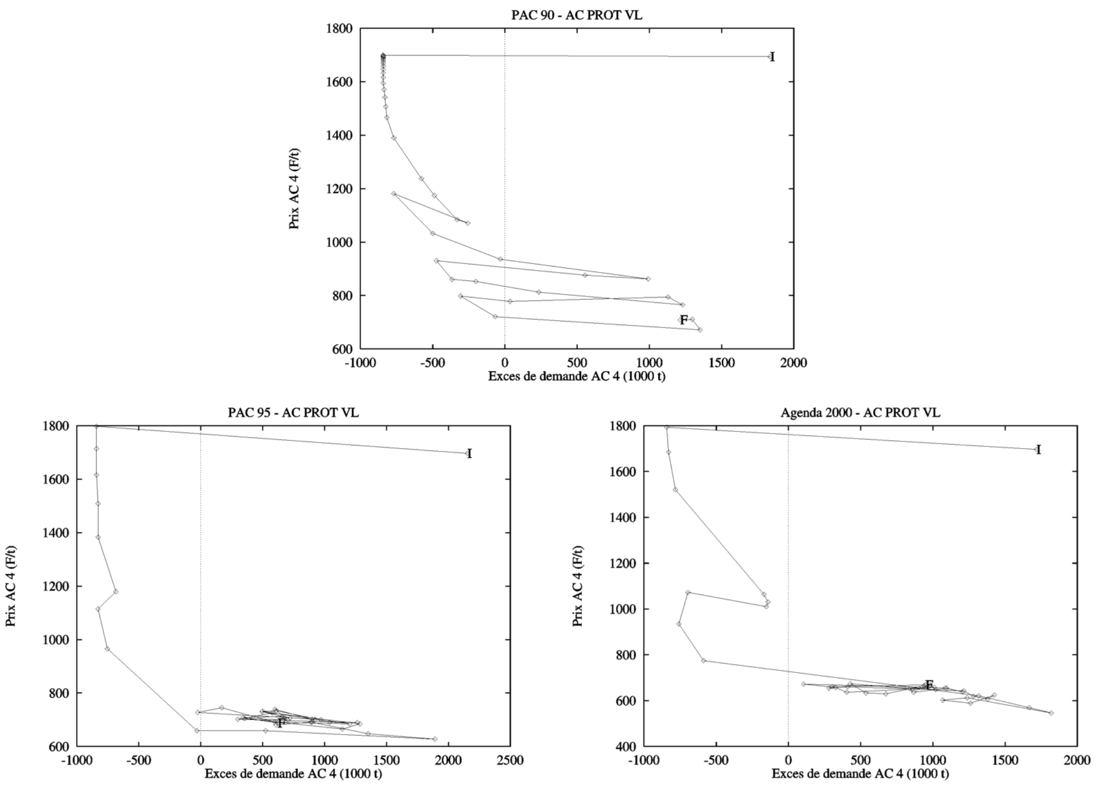

Figure 4. The market of feeds “proteins for beef catle”

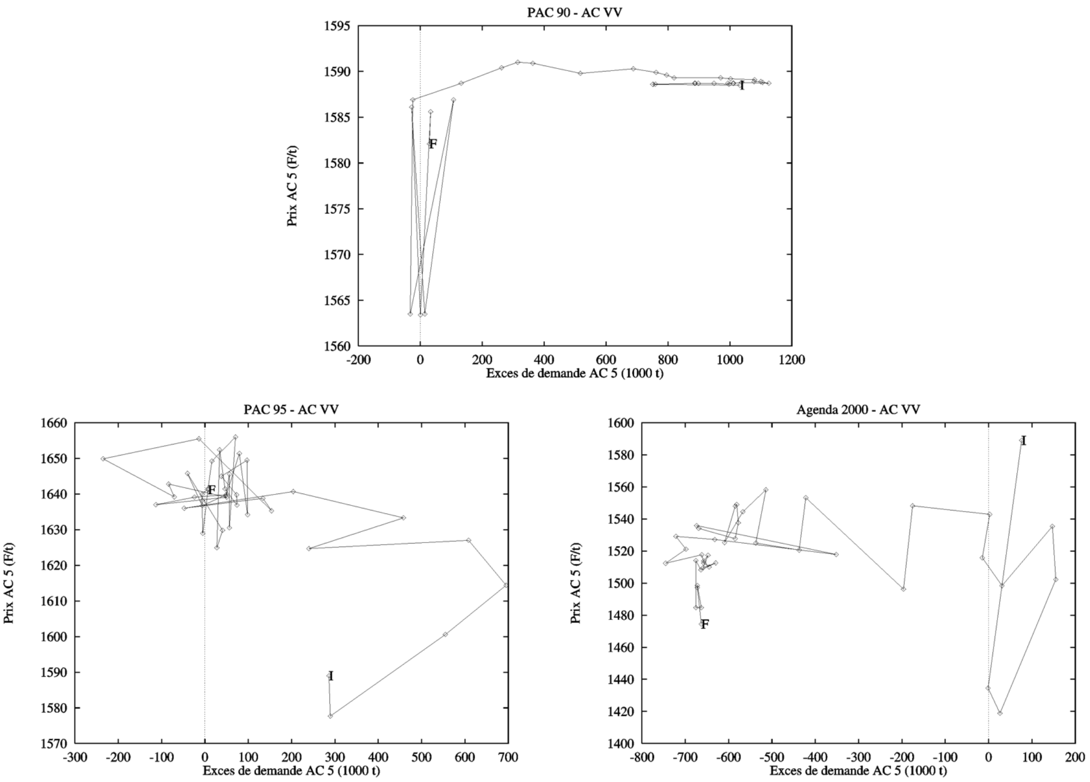

Figure 5. The market of feeds “concentratedfor beef cattle”

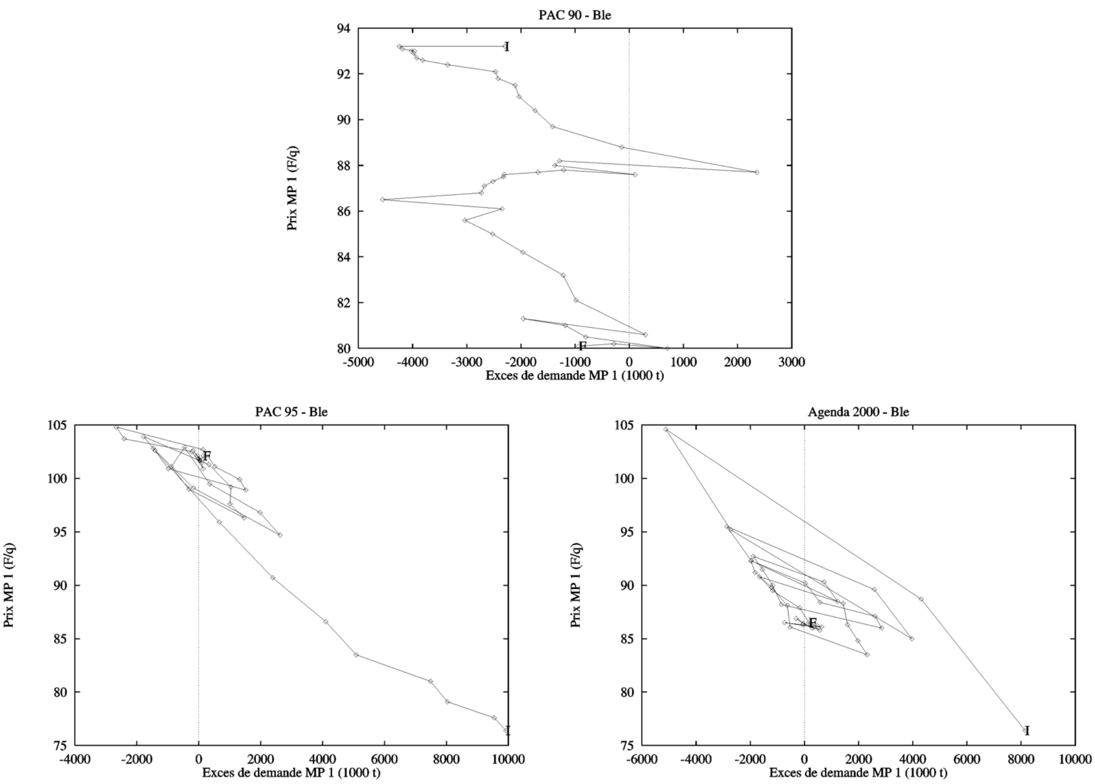

Figure 6. The market of soft wheat

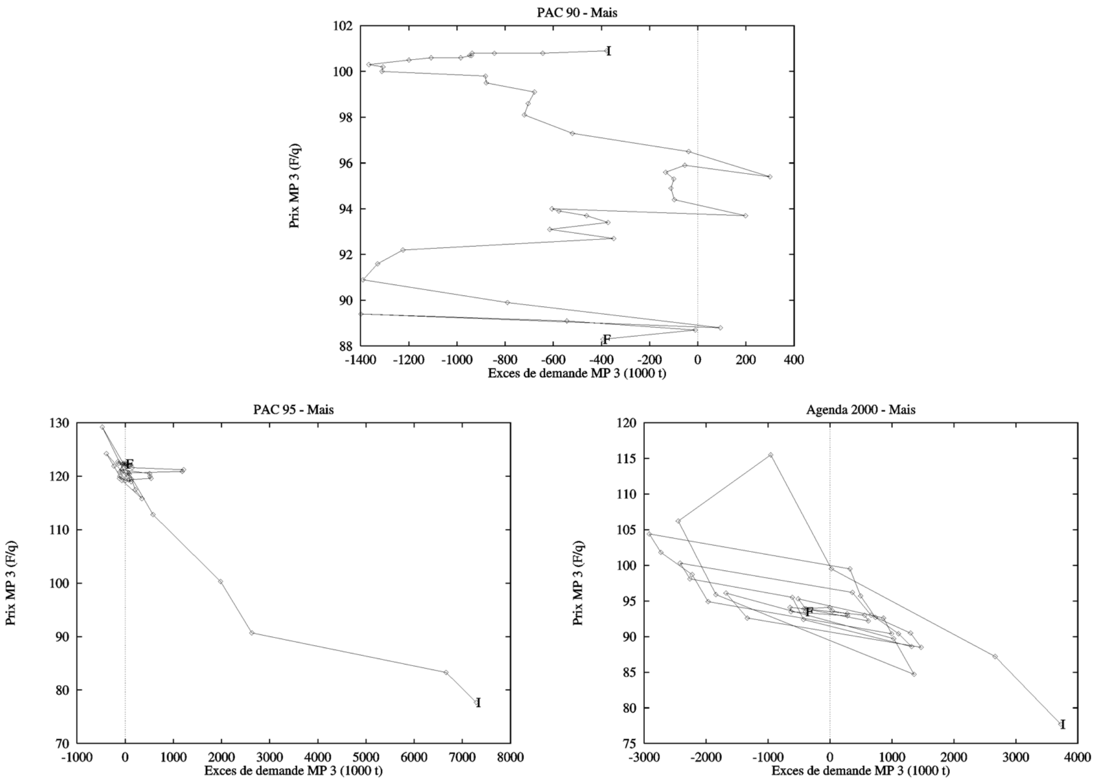

Figure 7. The market of grain corn

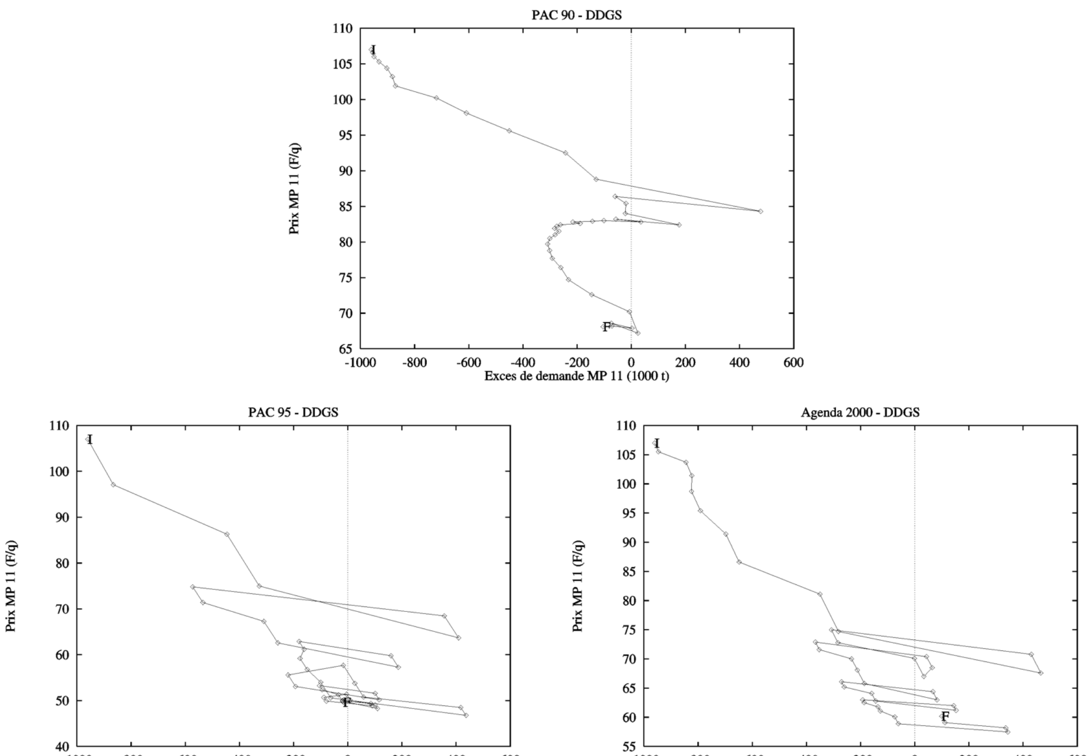

Figure 8. The market of "DDGS"

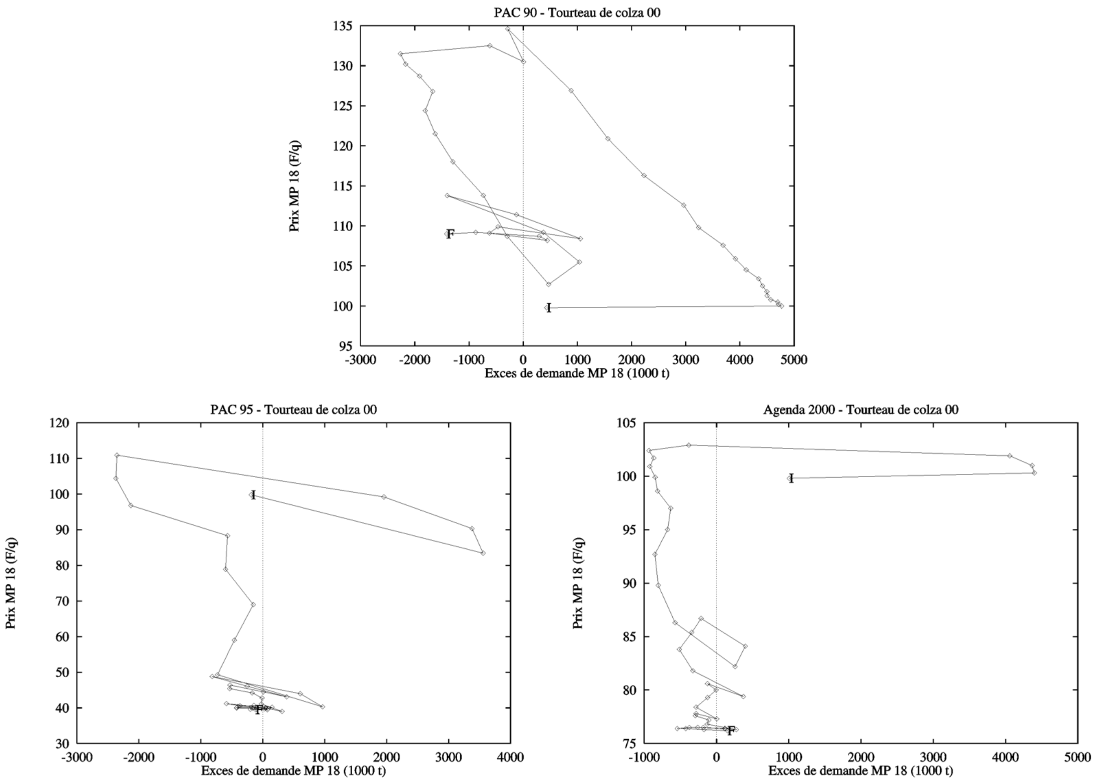

Figure 9. The market of rapeseed cakes

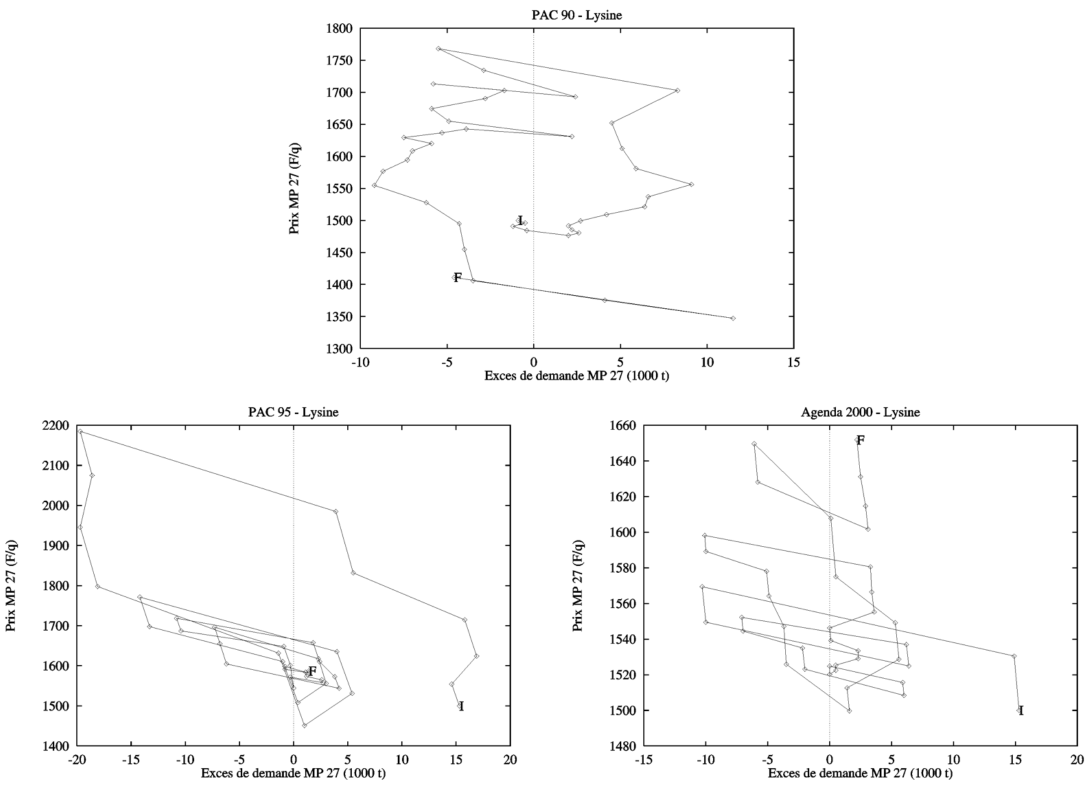

Figure 10. The market of lysin

More basic criticisms need, therefore, to be taken into account. Despite the assumption that the aggregation of the linear programs makes it possible to represent the sectors facing one another quite well one should ask oneself about the validity of the underlying implicit hypothesis of perfect concurrence. As far asthe agricultural producers are concerned, their large number is not incompatible with the idea of their affecting the products from outside the markets through lobbying. As for the livestock feed industrialists, it might be suggested that the model be run on the basis of oligopolistic behaviour given the small number of actors actually present in certain markets.

On the other hand, the large number of actors explicitly present in the models, at the expense of a typology and calibration that is both complex and demanding in data, makes it possible to counter-balance one of the habitual inconveniences of linear programming. Indeed, the characteristic stages of the supply and demand functions resulting from these models pose minor problems as they are numerous. One can thus explain why the model coupling the associated linear programs to the two sides of the markets makes it possible to converge towards simultaneous equilibrium in a large number of markets.

Appendix

The AROPAj model can be written as a set of linear programs in which x denotes the optimisation variable vector, p denotes the price vector and b denotes the right hand side vector. We define below some of the main components of these elements. For simplicity, we do not mention the index due to the farm typology.

$$\max \pi(x; p, A, b) = p_g.g + p_a.a - p_c.c$$

$$sc \left| \begin{array}{l} A_a \cdot s \leq 0 \\ g + f - A_y \cdot s \leq 0 \\ A_d \cdot a \leq b_a \\ A_l \cdot s \leq b_s \\ A_f \cdot \begin{bmatrix} c \\ f \end{bmatrix} - A_i \cdot a \leq 0 \\ CAP \end{array} \right.$$

Positive primal variables x are mainly detailed in g (the vector of crops gathering), f (the vector of on-farm consumptions), a (the vector of animal heads), c (the vector of bought feeds) and s (the vector of surfaces dedicated to crops, fodders, set-aside, and other).

Elements are composed of dierent sets. The objective price vector is denoted by p and prices or gross margins are detailed by indices such as g (crops), a (animal), c (bought feeds). A is the global matrix of elements with some sub-matrices such as A_a related to the 'agronomic' module, A_y the diagonal matrix of yields, Ad related to the 'demographic' cattle matrix, Ai denoting diagonal identity submatrices, A_l denoting 'line' submatrices, A_f related to the nutriment quality of feeds. We denote b the 'right hand side' (i.e. the resources vector) which can be detailed in b_s (total area), b_a (livestock capacity).

Numerous intermediate variables and elements, such as those attributable to CAP involvement, are not extensively given here. Let us just recall that the submatrix related to CAP is the most important one.

The MODANI model focuses on the sub-vectors g related to the aggregated cereal gathering supply and c related to the aggregated feed demand when aggregation sums up the farm type vectors. Let us denote the aggregated optimal activities in AROPAj that depend on the price p by $\hat{g}(p)$ and $\hat{c}(p)$ respectively.

The ALIMAj model optimizes the demand of raw materials and the supply of industrial feeds related to the regional transformation of cereals and other materials into concentrated feeds. Let us recall that ALIMAj calls for several linear programs, when one linear program is related to one region industry of transformation. We do not mention here the index related to the region number.

$$\max_x \pi(x; p, T, k) = p_c \cdot c - p_g \cdot g$$

$$sc \begin{vmatrix} T_l \cdot c \le k \\ c - T_g \cdot g \le 0 \end{vmatrix}$$

As in the previous case (see the AROPAj model presentation), the vector *p* denotes the objective price vector with its two components related to indices *g* (raw materials) and *c* (feed products). The primal variable vector *x* is too desaggregated in these two kind of activities.

The two kinds of constraints are first the capacity constraint (related to the upper limit *k*) involving the 'line' submatrix T_l, and, second the industrial transformation involving the submatrix T_g of raw materials contents required by the different kinds of industrial feeds.

The MODANI model focuses on the sub-vectors *g* related to the aggregated demand of cereals, and c related to the aggregated feed supply when aggregation sums up the regional vectors. Let us denote the aggregated optimal activities in ALIMAj and depending on the price *p* and the capacity *k* by $\overline{g}(p,k)$ and $\overline{c}(p,k)$ respectively.

The MODANI model deals with supplies and demands in a set of markets related to cereals and feedstuffs, which means that MODANI focuses on excess demand $\xi_c(p,k) = \overline{c} - \hat{c}$ and $\xi_g(p,k) = \overline{g} - \hat{g}$. Starting from initial values of the price and the industrial capacity, the computing process aims to lead to the price vector *p** which balances simultaneously the markets of raw materials and feeds ($\xi_c(p^*, k) = 0$ and $\xi_g(p^*, k) = 0$. Let us recall that the computation process is based on iterative tentatives in modifying prices according to the value of the vector of excess demand. It specifically assumes that industrial capacity can be changed between two iterations.

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