**Methods for risk assessment in water supply systems [Part 1. Components of drought planning. 1.3. Methodological component]**

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# Chapter 9. Methods for risk assessment in water supply systems

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## Introduction

The present chapter introduces concepts and methods related to risk and risk assessment of water shortages due to droughts in water supply systems. The main aim of the chapter is to provide methodologies able to quantify in a probabilistic way the risk of failure of a water supply system.

Two procedures for unconditional (planning) and conditional (operation) drought risk assessment of water supply systems are proposed. Both methodologies are based on Montecarlo simulation of the water supply system, in order to better take into account the stochastic nature of the hydrological input to the system.

The proposed methodologies results in an effective aid during both the planning and operating stages of a water supply system giving valuable information about expected frequency and amount of deficits related to the demands supplied by the system under study.

## Risk assessment in water supply systems

### General

Different definitions of risk are adopted in various disciplines, according to the objective of the analysis, as well as to the typology of event under study. Despite the differences, the several definitions can be broadly divided into two main categories: risk defined as the *probability of an adverse event*, and risk defined as the *expected (mean) consequence of an adverse event*. The first category includes the concept of risk according to statistical hydrology, defined as the probability that an hydrological variable $X$ (e.g. maximum annual discharge) exceeds a given threshold $x_o$ at least once in $n$ years:

$$\textbf{Risk} = P[\text{at least 1 year in } n \text{ years where } X > x_o] = 1\text{-}P[X \leq x_o \text{ in } n \text{ years}]$$

Assuming stationarity and independence of the events, the risk can be computed by the well known formula (Kottegoda and Rosso, 1998):

$$\textbf{Risk} = 1\text{-}P[X \leq x_o]^n$$

Similarly, in reliability theory, risk is defined as the probability of failure for the system under investigation. More specifically, risk is defined as the probability that the load $L$ (i.e. the external forcing factor) exceeds the resistance $R$ (an intrinsic characteristic of the system), leading to a failure (Mays and Tung, 1992):

$$\textbf{Risk} = P[L > R]$$

The second category (risk as expected consequence) includes the definitions developed within the strategies for natural disasters mitigation. In particular, risk is defined as "*the expected losses* due to a particular natural phenomenon as a function of natural hazard and the vulnerability of and element at risk" (UNDRO, 1991). In the above definition, the natural hazard represents the probability of

occurrence, within a specified period of time in a given area, of a potentially damaging natural phenomenon, whereas the vulnerability is the degree of loss to a given element at risk or set of such elements resulting from the occurrence of a natural phenomenon of a given magnitude and expressed on a scale from 0 (no damage) to 1 (total loss). It follows that according to the above definition, risk is measured in some physical terms, such as economical (damages) or social (lives lost). Also, such risk definition has found widespread application in flood analysis, since it is particularly suited for the development of inundation risk maps in a given area (Kron, 2005).

When dealing with drought risk in water supply systems characterized by a high level of complexity and interactions among the different components, it is easy to recognize that none of the above definitions is able to encompass the different dimensions and consequences related to water shortages. Therefore, traditionally, characterization of the shortages in a water system has been carried out by means of a set of performance indices, attempting to capture different aspects related to concepts such as reliability, resiliency and vulnerability (Hashimoto *et al.*, 1982). Indeed, stochastic nature of inflows, high interconnection between the different components of the system, presence of many sometimes conflicting demands, definition of the elements at risk, uncertainty related to the actual impacts of extreme events such as droughts, make the risk assessment of a water supply system a problem that is better faced through a set of several indices and/or by analyzing the probabilities of shortages of different entities (Alecci *et al.*, 1986).

With regard to the *risk analysis* it is generally recognized that it can be divided into *risk assessment* and *risk management*. The former is oriented to the estimation of the probabilistic features of an adverse phenomenon, whereas the latter is generally defined as a pro-active approach for coping with risk through planned actions, as apposite to crisis or emergency management. Risk assessment therefore has the objective to quantify probabilistically the occurrence of an adverse phenomenon, as well as to estimate its consequences. Risk management has the objective to identify in advance a set of measures oriented to prevent or to mitigate consequences of the adverse phenomenon.

Risk assessment can find application either at the planning stage or during the operation of a given system. For instance, with reference to water supply system planning, risk assessment enables to quantify and compare the risk associated with different planning alternatives, generally on a long term basis. On the other hand, during the operation of the system, short term drought risk assessment can be carried out in order to compare and define alternative mitigation measures, on the basis of the consequent risk during a short time horizon (e.g. 2-3 years) in the future. The two approaches differ, not only with regard to the objective of the analysis and to the different lengths of the time horizons, but mostly because of the way the probabilistic assessment is carried out. In the first case, the assessment is generally *unconditional*, i.e. without regards to the initial state/condition of the system and therefore it provides information on what could happen at any time during the planning horizon. For instance, with reference to a water demand, one may be interested in the probability of occurrence of a given deficit during the planning horizon. The short term risk assessment, on the other hand, is generally *conditional*, in the sense that the initial state/conditions of the system are taken into account in the evaluation. Furthermore, the assessment is generally oriented to estimating what could happen at a specific time in the immediate future. For instance, with reference to a water use, one may be interested in the probability of occurrence of a given deficit three months ahead, given the present state of the system (e.g. volumes stored in reservoirs). As a such, the conditional assessment is generally adopted for early warning purposes. Since the results of the conditional risk assessment strongly depend on the initial conditions, it follows that the procedure must be repeated as new information becomes available.

## Unconditional (long term) risk assessment

Unconditional risk assessment has the objective of the comparison and the selection of preferred drought mitigation alternatives through the simulation over a long time horizon (30-40 years) of the system behaviour by using generated series. Then, the risk is evaluated in terms of a synthetic assessment of failure based on the analysis of the satisfaction of consumptive demands (both in time and volume), as well as of meeting some specified objectives such as the satisfaction of ecological requirements, or target storages in reservoirs.

The term *unconditional* here refers to an assessment without regards to the initial state/condition of the system, and therefore the procedure is oriented to provide information on what could happen at

any time during the explored planning horizon. To achieve the above objective the study can start at any initial condition of the system because this will be irrelevant to the overall behavior of the system during a long time horizon.

Figure 1 shows the proposed methodology for unconditional drought risk assessment for a water supply system. The procedure is divided into three main tasks, namely system identification, hazard analysis and risk assessment. The system identification tasks consists in the definition of all the relevant information regarding the water supply systems, with reference to the hydrological inputs, the physical features of the elements of the system and the different uses/sectors as well as their water demands and historical used volumes.
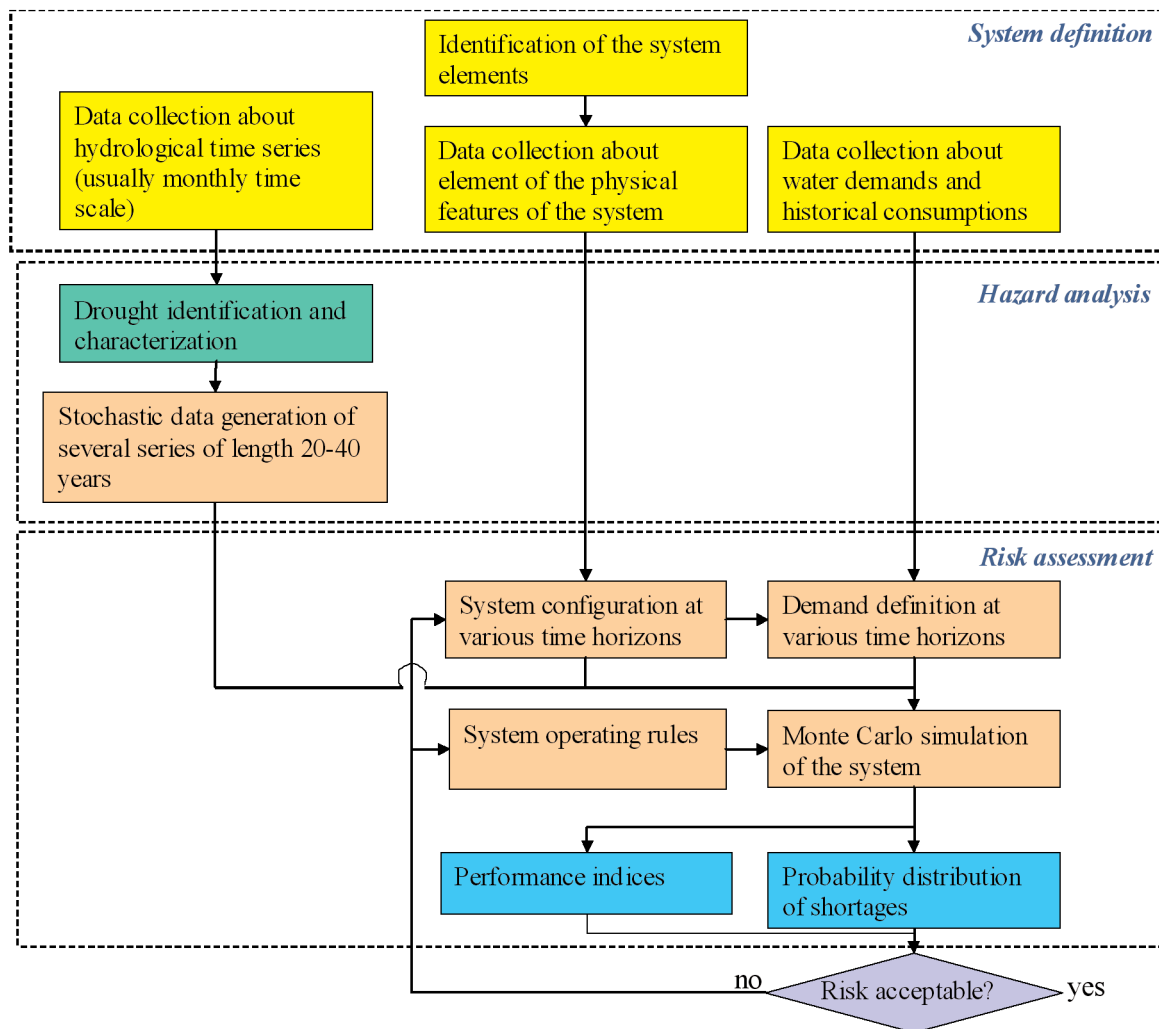


Fig. 1. Methodology for unconditional drought risk assessment in water supply systems planning.

Then, a hazard analysis is carried out, with the objective to characterize probabilistically the drought features. Such characterization can be performed by estimating the return period of droughts of different severities, by means of the methodologies presented in the Chapter on Drought Characterization of this volume, and implemented in the software REDIM.

Within the risk assessment task, one of the primary objective is evaluating the system state variables and other variables related to the satisfaction of various demands (e.g. water supply shortages) under a given system configuration and a given set of operating rules by considering, as hydrological input, several generated streamflow series. Furthermore, a similar assessment is also required for the satisfaction of ecological requirements, such as instream flow requirements, and for

target storages in reservoirs. Synthetically generated series can be obtained by means of a stochastic model fitted to the observed series, such that the generated series resemble, in a statistical sense, the observed ones. Thus, each generated series can be considered as one of the possible series that will occur in the future and, as a consequence, the data resulting can be seen as a large sample from the population of all the possible system behaviors in the future (Montecarlo simulation). Then, probabilistic features of the consequences of drought can be assessed by performing a statistical analysis of the results of simulation.

The results of the Montecarlo analysis enable to verify whether the system exhibit an acceptable risk of shortage under the given configuration and set of operating rules. If this is not the case, the procedure can be repeated by analyzing different configurations and/or operating rules.

## Conditional (short term) risk assessment

The proposed procedure for conditional (operational) risk assessment has the objective of evaluating of the risk of shortages in a short time horizon by using generated series. The procedure makes use of the same basic tools (namely stochastic data generation, water system simulation and synthetic assessment of performance), but in this case the analysis is performed with reference to a shorter time horizon (2-3 years) and by taking into account the initial state/conditions of the system. Thus, the results will depend on when the analysis is performed, since they will change as new information is available. Therefore, such procedure should be carried out at given time steps (e.g. every month) during the operation of the system, in order to identify potential failures in the future and to implement the necessary measures.

Different criteria could be applied to decide the length of time horizon for conditional risk assessment of a given system. In particular it should be defined taking into account the length of historic droughts, consolidated operating procedure of the system, time horizon prescript by the law, the need to avoid the growth of evaporation losses caused by a pluriannual management of reservoirs.

With reference to the scheme depicted in Fig. 2, the system identification will include the monitoring of current meteo-hydrological conditions, of the storage volumes in the reservoirs and the definition of water demands. Then an hazard analysis is carried out in order to characterize probabilistically the current drought conditions. Again, such characterization can be performed in terms of return periods of droughts identified on streamflow series. The first step of the risk analysis is carried out by generating several series over a short time horizon (2-3 years), conditioned on the hydrological observations up to the moment when the analysis is performed. Then, the system is simulated, by assuming as initial conditions (e.g. volumes in reservoirs) the actual ones when the analysis is carried out. Thus the risk assessment will enable to estimate the risk at specified intervals in the immediate future (e.g. 1 month, 2 months, etc.) since such conditional risk is strongly affected by the initial conditions.

Application of the proposed methodology enables to assess, in a probabilistic way, the short term risk of failures considering the actual condition of the system, thus giving the opportunity to explore effects of different policies of management and mitigation measures.

## Tools

## Simulation of water supply systems

Simulation has the objective of reproducing the real world based on a set of assumptions and conceived models of reality (Ang and Tang, 1984; Labadie, 2004). Because the purpose of a simulation model is to duplicate reality, it is a useful tool for evaluating the effects of different designs, hydrology, mitigation measures against drought and/or operating policies on system performances.

Simulation models are perhaps the most widely studied and applied methods for analyzing and evaluating alternatives to manage water supply systems. The reason for their popularity lies in the fact that such models can approximate very closely the systems using a relatively simple mathematics and are easily understood by water managers. Water supply systems are generally *complex systems* in which the components (e.g. reservoirs, diversions, etc) are arranged as a mixture of in-series and in-

parallel, or in the form of a loop. When dealing with a complex system, the general approach is to reduce the system configuration, based on its component arrangement or modes of operation, to a simpler system for which the analysis can be performed easily.
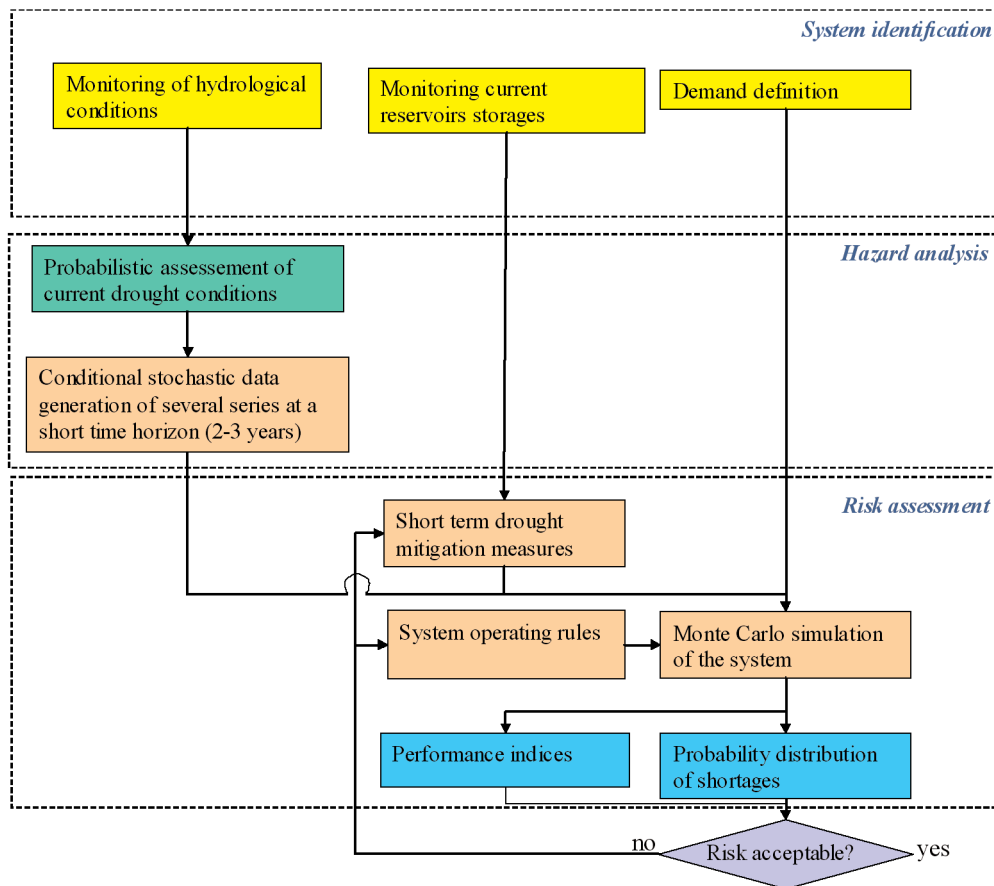


Fig. 2. Methodology for conditional drought risk assessment in water supply systems operation.

Any simulation model is typically based on mass balances of water quantities in the components constituent the whole system. The system dynamics equations are generally based on preservation of conservation of mass throughout the system and following a node-arch approach to describe the system network, and can be written as follows:

$$\mathbf{S}_{t+1} = \mathbf{S}_t + \mathbf{C}.\ \mathbf{r}_t + \mathbf{q}_t - \mathbf{l}_t\ (\mathbf{S}_t, \mathbf{S}_{t+1}) - \mathbf{d}_t \text{ for } t = 1, ..., T$$

where $\mathbf{S}_t$ = storage vector at the beginning of time t; $\mathbf{q}_t$ = inflow vector during time t; $\mathbf{C}$ = system connectivity matrix mapping flow routing within the system; $\mathbf{r}_t$ = downstream releases from reservoirs or diversion points; $\mathbf{l}_t$ = vector combining spills, evaporation, and other losses during time t; and $\mathbf{d}_t$ = releases from the system to satisfy demands and or water transfers. Calculation of evaporation and other water losses in the term $\mathbf{l}_t(\mathbf{S}_t, \mathbf{S}_{t+1})$ is usually difficult to evaluate correctly, and therefore approximations are generally adopted. All flow units are expressed in storage units per unit time. Spatial connectivity of the water system network can be fully described by the routing or connectivity matrix $\mathbf{C}$ having 1 in the *i,j* elements to connect node *i* to node *j* and 0 otherwise (Labadie, 2004).

The output of a simulation model include the series of releases to the water users, the series of volumes stored in reservoirs, as well as other information such as downstream releases, withdrawals from marginal resources, etc. Thus, for any set of design and operating policy parameter values, simulation provides a rapid means for evaluating the anticipated performance of a system. Simulation models do not identify optimal operating policies but they are an excellent aid to water managers in evaluating effects on the system, including risk of drought, of different alternatives (planning) or given mitigation measures and/or operating policies (operation).

Critical issues for simulation models are the definition of the boundaries of the system that is to be simulated, the level of detail within the system that should be modeled and the time scale. Furthermore there are difficulties associated with sampling in the multidimensional space which contains the vector of the operating decision variables (Mays, 1996).

Simulation models have to be able to be connected to other models (i.e. stochastic generation models); they have to be general but enough versatile to simulate peculiar features and operating conditions of virtually any system. Furthermore they have to be easy to use and to understand in order to be accepted both by decision makers and end-users making really effective the proposed mitigation measures, operating rules and/or procedures to cope with risk.

In the case of water supply systems, simulation models have mainly to help with: choice of supplies, connections between elements of the system, withdrawal order from sources in order to satisfy demand patterns and, in the case of shortages, assessment of their distribution in time and among the different users. Furthermore they have to be able to evaluate actual effectiveness of proposed mitigation measures, helping to define triggers to activate operating policies and giving results in a comprehensive manner.

Simulation models can be *time-sequenced* or *event-sequenced, deterministic* or *stochastic,* dealing with *steady-state* or *transient* conditions (Loucks, 1981). The model to use in the proposed methodology should be a time-sequenced able to deal with transient conditions, that is implementation of different alternatives for the planning (e.g. unconditional risk assessment where both changes in configuration and in operating rules must be taken into account during the simulation time horizon).

Simulation models can effectively be used to manage a complex system on a continuous basis but also to manage extreme events such as drought that occur over a relatively short-time horizon. These two different types of model applications will require models having different temporal and/or spatial resolutions. Planning models are used sequentially but, being the time horizon longer than operating models, the interest is focused on the overall behavior of the system including major changes in its configuration to compare different scenarios.

Operating models are still used sequentially. They need to be continually updated and rerun to get the most current estimates of what operating decisions should be made for each component constituting the whole system in each future decision period.

Some of the most important simulation models are HEC-PRN (Hydrologic Engineering Center, 1993), AQUATOOL (*Andreu et al.*, 1996), MODSIM (Labadie *et al.*, 2000), STELLA (Stein *et al.* 2001), VENSIM (Caballero *et al.*, 2001), POWERSIM (Varvel and Lansey, 2002).

Simulation models or descriptive models are surrogate for asking "what-if" questions regarding the performance of alternative operational strategies. They can accurately represent system operations and are useful for Montecarlo analysis in examining long or short-term reliability of proposed operating strategies.

Simulation models of water resources systems, whether used for planning or for operating management, merely provide information. Actual decisions need still to be taken by water managers using models as aids in order to make "informed" decisions. In order to be well accepted by water managers and thus really effective for real cases, models have to be as much versatile as possible offering a range of not prescriptive alternatives. Models cannot determine which assumptions and data are best, they can only help identify the impacts of those assumptions and data (Mays, 1996).

## Synthetic series

Because of the stochastic nature of the hydrological inputs to water supply systems, Montecarlo simulation results is a powerful tool to cope with uncertainty affecting risk assessment both in planning and operating stage. In order to perform Montecarlo analysis, an appropriate stochastic model must be selected for generating numerous synthetic hydrological series that preserve some statistical properties of historical series.

The general aim of a stochastic model is to reproduce as closely as possible the true marginal distribution of seasonal and/or annual hydrological variables. Also, modeling the joint distribution of flows at a different site in different months, seasons, and years may be required for multicomponent water supply systems. The persistence of flows often described by their autocorrelation is also another important aspect, since it affects the reliability with which a reservoir of a given size can provide a specific yield.

Several models have been developed with the aim of preserving one or more characteristics of investigated series. They usually differ according to the time scale of the analysis, since for instance in the case of data aggregated at sub-yearly time scale the seasonality of the statistics must be taken into account. Accordingly, models can be stationary or periodic. Models can also be classified according to whether the interest lies in modeling one series (univariate models) or several series jointly preserving the cross correlation (multivariate models). Also, while most models are developed in the normal domain thus requiring a preliminary data transformation, in the case of non-normal observations some models are able to generate directly skewed data (Salas, 1993).

One of the most widely used stochastic model is the AR(p) model that can be written as follows:

$$y_t = \mu + \sum_{j=1}^{p} \phi_j(y_{t-j} - \mu) + \epsilon_t$$

where $y_t$ is the stochastic variable to be modeled, $p$ is called order of the model while $\epsilon_t$ is a normally distributed uncorrelated random variable called *noise*, *innovation*, *error term*, o *series of shocks* with mean zero and variance $\sigma^2_\epsilon$ and uncorrelated with the $y_t$ process.

Since $\epsilon_t$ is normally distributed then also $y_t$ is normal. Model parameters are $\mu$, $\phi_{1...}$ $\phi_p$ and $\sigma^2_\epsilon$. Lower order models, with $p = 1, 2$ or 3 have been widely used to generate synthetic annual series.

The simplest model, AR(1) can be written as:

$$y_t = \mu + \phi_1(y_{t-1} - \mu) + \varepsilon_t \qquad (1)$$

with mean and variance:

$$E[y]_t = \mu \qquad Var[y] = \sigma^2 = \frac{\sigma^2_\varepsilon}{1 - \phi_1^2}$$

while the autocorrelation function is:

$$r(k) = \phi_1^k$$

A more versatile model than the AR(p) is the *autoregressive moving average model* ARMA(p,q) with *p* autoregressive parameters and *q* moving average. Using the same notation adopted in (1) an ARMA(p,q) model can be written as follows:

$$y_t = \mu + \sum_{j=1}^{p} \phi_j(y_{t-j} - \mu) + \epsilon_t - \sum_{j=1}^{q} \theta_j \epsilon_{t-j}$$

A simple version of the ARMA(p,q) model is the ARMA(1,1) as:

$$y_t = \mu + \phi_1(y_{t-1} - \mu) + \varepsilon_t - \theta_1 \varepsilon_{t-1}$$

which mean and variance are:

$$E[y] = \mu \qquad Var[y] = \sigma^2 = \frac{\sigma^2_\varepsilon}{1 - \phi_1^2}(1 - 2\phi_1\theta_1 + \theta_1^2)$$

where $\phi_1$ is:

$$\phi_1 = \frac{r_2}{r_1}$$

and $\theta_1$ is a function of $\phi_1$ and $r_1$.

When the original series is characterized by strong seasonality PAR($p$) and PARMA($p,q$) (*periodic autoregressive model* e *periodic autoregressive moving average model*) are able to reproduce this character.

Assuming that a periodic hydrological process is represented by $y_{nt}$, in which $n$ defines the year and $t$ defines the season, such that t = *1, …, w* and w is the number of season in the year (seasons, months, weeks) a PAR($p$) model is defined as follows:

$$y_{\nu,\tau} = \mu_\tau + \sum_{j=1}^{p} \phi_{j,\tau}(y_{\nu,\tau-j} - \mu_{\tau-j}) + \epsilon_{\nu,\tau}$$

in which the meaning of the symbols is the same of that given before for the AR($p$) and ARMA($p,q$) models with the difference that this time the parameters of the model to be estimated are $\mu_t$, $\phi_{1,\tau}$, …,$\phi_{p,\tau}$ and $\sigma^2_\tau(\varepsilon)$ for t = 1,…, w.

By considering a moving average component, a PAR($p$) becomes a PARMA($p,q$) model, that can be written as follows:

$$y_{\nu,\tau} = \mu_\tau + \sum_{j=1}^{p} \phi_{j,\tau}(y_{\nu,\tau-j} - \mu_{\tau-j}) + \epsilon_{\nu,\tau} - \sum_{j=1}^{q} \theta_{j,\tau}\epsilon_{\nu,\tau-1}$$

When synthetic data generation models are used in a Montecarlo simulation of water supply system with several hydrological input, it is generally necessary to generate cross-correlated series that preserve also the correlation between the different inflows within the same water supply system. Formulation of this kind of models is similar to the one shown for AR($p$) and ARMA($p,q$) models with the difference that a matrix notation is now needed. Specific models such as MAR($p$) and MARMA($p,q$) (*multivariate autoregressive models* and *multivariate autoregressive moving average models*) are useful for this task.

Consider a multiple time series **Y**, a column vector with elements $y_t^{(1)}$, …, $y_t^{(n)}$ in which $n$ is the number of series (number of sites or number of variables) under consideration. The multivariate MAR(1) model is defined as:

$$\mathbf{Z}_t = \mathbf{A}_1 \mathbf{Z}_{t-1} + \mathbf{B}\varepsilon_t$$

in which $\mathbf{Z}_t = \mathbf{Y}_t\text{-m}$, $\mathbf{A}_1$ and **B** are $n \times n$ parameter matrices and **m** is a column parameter vector with elements $m^{(1)}$, …, $m^{(n)}$. The noise term $e_t$ is also a column vector of noises each with zero mean, uncorrelated with $\mathbf{Z}_{t-1}$ and normally distributed.

Using the same notation MARMA($p,q$) models can be introduced. The simplest MARMA($p,q$) is the MARMA(1,1) that can be defined as:

$$\mathbf{Z}_t = \mathbf{A}_1 \mathbf{Z}_{t-1} + \mathbf{B}\varepsilon_t - \mathbf{C}_1\varepsilon_{t-1}$$

in which $\mathbf{C}_1$ is an additional $n \times n$ parameter matrix useful to consider the moving average component of the original series.

Using the full MAR($p$) and MARMA($p,q$) models often leads to complex parameter estimation, thus some model simplifications have been suggested. For instance a simpler model considers $\mathbf{A}_1$ to be a diagonal matrix. In general a *contemporaneus* ARMA($p,q$) (CARMA) model results if the matrices $\mathbf{A}_p$ and $\mathbf{C}_q$ are considered to be diagonal. In this case the model implies a contemporaneous relationship in that only the dependance of concurrent values of the $y$'s are considered important.

Skewed hydrological processes must be transformed into normal processes before AR or ARMA models are applied. However, a direct modeling approach which does not require a transformation may be a viable alternative. For instance, the *gamma autoregressive process* offers such an alternative. It is defined as:

$$y_t = \phi(y_{t-1}) + \varepsilon_{\gamma t}$$

where $\phi$ is the autoregressive coefficient, $(\varepsilon_t)$ is a random component that can be obtained as a function of $f$ and the parameters of a Gamma distribution (location, scale, shape).

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Stochastic data generation models are often said to statistically resemble the historic flows if the model produces synthetic flows with the same mean, variance, skew coefficient, autocorrelation, and/or cross-correlation as in the historic series. The drawback of this approach could be that it shifts the modeling emphasis on reproducing arbitrarily selected statistics of the available data. Therefore, for any particular water supply system, and depending on the purpose of the analysis one must determine what particular characteristics need to be modeled. Such decision should depend on what characteristics are important to the operation of the system being studied as well as on the data available.

## Analysis and representation of results

The output of Montecarlo simulation of a water supply system consists in several series of storage levels in reservoirs, downstream releases, releases to the demands, etc. Analysis of such results therefore must be carried out by means of synthetic indices, able to capture different features of the analyzed series. Here, for the purpose of risk analysis of water shortages due to droughts, the following synthetic assessment of system failures in terms of satisfaction of consumptive demands are proposed:

(i) Water supply system performance indices (reliability, resilience and vulnerability)

(ii) Frequency plot of shortages

(iii) Histogram of monthly frequencies of shortages

(iv) Sample frequency of monthly shortages

(v) Return period of shortages defined as the average interarrival time between two annual shortages exceeding a given value

A similar assessment can be proposed for the satisfaction of ecological requirements, such as instream flow requirements, and for target storages in reservoirs.

Some of the most meaningful water supply system performance indices are:

(i) Temporal reliability

(ii) Volumetric reliability

(iii) Average shortage period length

(iv) Max monthly shortage

(v) Max annual shortage

(vi) Sum of squared shortage

Temporal reliability is defined as the probability that the system is in a satisfactory state.

$$Aff_t = \Pr[X_t \in S]$$

where $X_t$ represent the state of the system at time $t$ and $S$ is the ensemble of the satisfactory states.

If by satisfactory state we indicate the complete meeting of demands, this probability can be estimated as the ratio between the number of intervals during which demand is fully met and the total number of intervals considered.

$$rel_T = \frac{n_s}{N}$$

where $n_s$ is the number of intervals during which demand is fully met and $N$ is the total number of intervals considered. This index gives information about the time reliability of the system on a given demand.

Volumetric reliability is expressed as the ratio between the total volume released and the total demand volume:

$$rel_V = \frac{\sum\limits_{t=1}^{N} R_t}{\sum\limits_{t=1}^{N} D_t}$$

where $R_t$ and $D_t$ are respectively the volume released and the demands at the $t$ interval. This index helps on the evaluation of the total volumes released by the system.

The average shortage period length is defined as:

$$Av_{def} = \frac{N - n_s}{N_P}$$

where $n_s$ is the number of intervals during which demand is fully met, $N$ is the total number of intervals considered and $N_p$ is the number of periods of deficit defined as a continuous series of deficit intervals.

The maximum monthly and annual shortage are defined as the maximum of the annual and monthly shortages series and gives information about the vulnerability of the system in a single interval.

The sum of squared shortages index gives information about the amount of the shortages.

The above mentioned indices give an objective estimation of performance of the system but are not sufficient to catch some interesting statistical features of the shortage series.

Histogram of monthly frequencies of shortages, sample frequency of monthly shortages and return period of shortages defined as the average interarrival time between two annual shortages exceeding a given value, expressed in form of graphs, can help to capture the stochastic features of shortages.

In particular histogram of monthly frequencies of shortages, as depicted for example in Fig. 3, represent the frequency of shortages belonging to one of the four proposed classes expressed as a percent of the demand of a given interval (0-25%, >25%-50%, >50%-75%, >75%-100). This representation gives information about the overall monthly probability of deficits and their distribution among the classes.

Sample frequency of monthly shortages, as depicted for example in Fig. 4, represents non exceedence probabilities of shortages giving the opportunity to estimate the frequency of shortages of different entity as a continuous curve.

Return period of shortages, defined as the average interarrival time between two annual shortages exceeding a given value, gives information about the rarity of the shortages.

An example of the comparison of return period of shortages for two different operations of the system (with or without mitigation measures) is depicted in Fig. 5. From the figure, it can be inferred how the return period of dimensionless shortages greater than 0.3 when mitigation measures are applied is larger than the corresponding return period when no measures are applied. Thus it can be concluded that the adoption of the measures is beneficial for dimensionless shortages greater than 0.3 since the interarrival time increases significantly.

Comparison between the above mentioned indices and graphs calculated for simulations done in correspondence of different implemented mitigation measures can help on evaluating in a statistical sense the impacts of mitigation measures on different demands relying on the system under investigation.

Even if it is not possible to define a unique synthetic index to assess the risk of a given water supply system, however an analysis based on the mentioned indices and graphs can give a good idea of the multifaceted behavior of a water supply system under drought conditions.
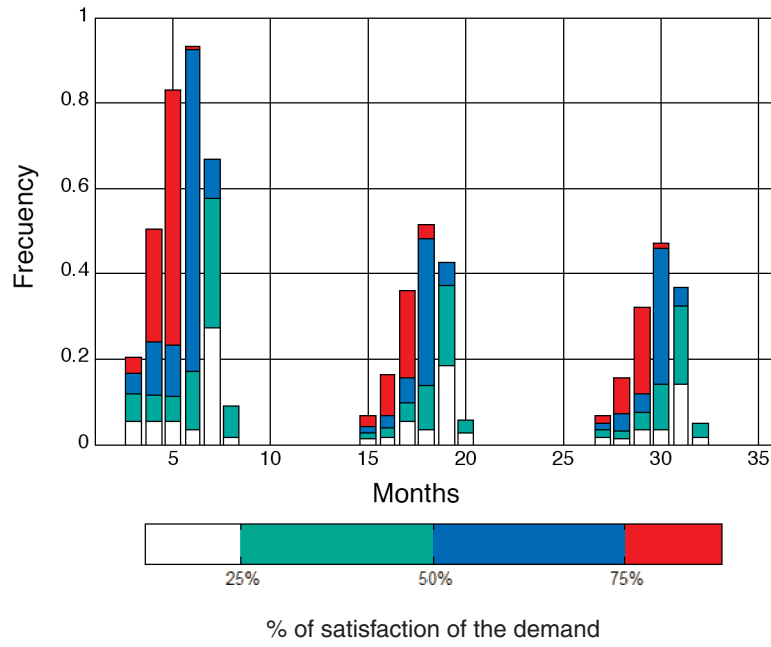
Fig. 3. Example of histogram of monthly frequencies of shortages in percentaje of demand.
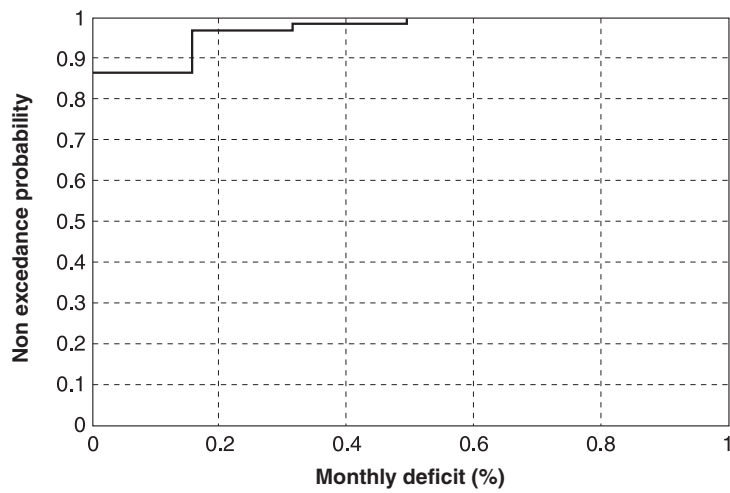


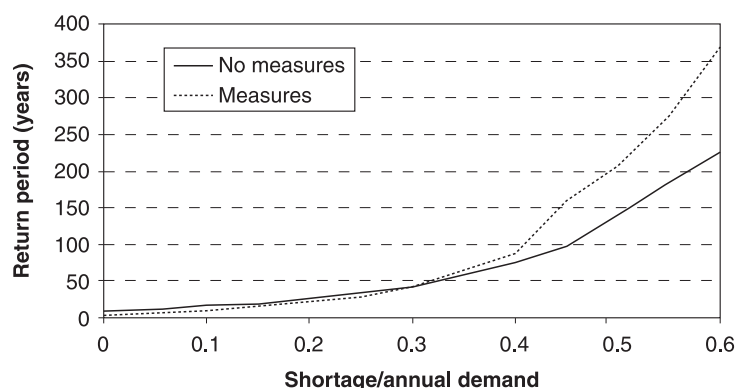Fig. 4. Sample frequencies of monthly shortages.

Fig. 5. Comparison of return period of shortages in two different operating modes of the system.

## Conclusions

Despite several definitions of risk exists, there is a general agreement that dealing with risk is basically dealing with the uncertainty of the consequences of a given phenomenon. Such uncertainty stems from the stocasticity that characterize most of the natural phenomena, as well as from the difficulties in assessing in a deterministic way their consequences and impacts. When assessing drought risk for a water supply system, to the above difficulties must be added also the fact that the same drought can have different consequences on the same system, depending on the degree of preparedness (i.e. mitigation measures) of the system.

Therefore, a correct approach to assessing risk in water supply system must be based on tools able to fully capture the stochastic nature of the drought phenomenon, as well as to evaluate the effects of different management alternatives of the system. Within this framework, Montecarlo simulation represents the ideal tool, since it enables to overcome the limitations of a probabilistic evaluation of risk of shortage based on historical hydrological series, which is hindered by the generally limited sample length availability. Simulation of the system using synthetically generated series also enables to extend the analysis, besides the planning stage, also during the operation of the system, by assessing the conditional risk, i.e. the risk of shortages in a short term time horizon as a function of the current states of the system. Furthermore, an appropriate analysis of the results of Montecarlo simulation allows to capture the multifaceted features of water shortages, thus allowing for an improved assessment of the impacts of droughts to be carried out.

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