The application of optimisation techniques to water resources problems

Aït-Kadi M.

Etat de l'agriculture en Méditerranée : Ressources en eau : développement et gestion dans les pays méditerranéens

Bari : CIHEAM
Cahiers Options Méditerranéennes; n. 1(1)

1993
pages 143-158

THE APPLICATION OF OPTIMIZATION TECHNIQUES TO WATER RESOURCES PROBLEMS

M. Ait Kadi (*)

INTRODUCTION

The need to conserve and manage water is not new, although in the past most countries could meet their requirements for domestic, industrial and agricultural supplies well within the water resources available to them. As population numbers increase, coupled with demands for high per capita domestic and industrial consumption resulting from improved standards of living, the sustainable upper limit or "carrying capacity" of water resources utilisation will be approached very rapidly over the next two or three decades in many countries. The situation is most acute in countries which are already heavily dependent on irrigation to meet their domestic food needs. This impeding crisis is unlikely to be recognised as an absolute resource constraint unless planning methods are adopted which are designed to investigate water resources in a comprehensive manner. Hence, in the context of the current critical water shortages and the serious and growing threat they pose to sustainable development and protection of the environment, the Dublin Statement calls for "fundamental new approaches to the assesment, development and management of fresh water resources, which can only be brought about through political commitment and involvement from the highest levels of government to the smallest communities. Commitment will need to be backed by sustainable and immediate investments, public awarness campaigns, legislative and institutional changes, technology development, and capacity building programs...". Underlying all this is the recognition that current water resources planning and management procedures are inadequate to address either the problems of impeding critical water shortages or current concerns about sustainability. There is therefore a felt need for a fundamental reassesment of the methods of analysis and water management adopted and the development of a new framework and new techniques.

Well, during the last 20 years, one of the most important advances made in the field of water resources engineering is the development and adoption of optimization techniques for planning, design, and management of complex water resources sytems. Extensive literature review of the subject reveals that many succesful applications of these techniques have been made. Therefore, the use of optimization techniques for water resources systems planning and management is not new; what is, new however, is the need to make these methods more transparent so that they can find widespread acceptance among all those concerned by the rational development and use of increasingly scarce water resources.

This paper is not intended to be a review and evaluation of the state-of-the-art of optimization techniques applied to water resources systems. Rather it is intended to introduce readers to some of the simplest optimization techniques that have seen successful field applications. The tone of the paper reflects our conviction that courses in these techniques should become commonplace in our training programmes as they can significantly contribute to the needed cultural for a better management of our nations' most vital resources. It is hoped that it will extend the use

(*) Professor, Département de l'Equipement et de l'Hydraulique, Institut Agronomique et Vétérinaire Hassan II, BP 6202, Rabat instituts, Maroc

of optimization techniques to a wider audience concerned with water resources planning and management. However, the application of mathematical optimization techniques should not occult the complexity of the multiple demands now being made upon our scarce fresh water resources and the far-reaching impacts which many water use activities are having. This forces us to consider water management in fully comprehensive manner.

ON THE COMPLEXITY OF WATER RESOURCES MANAGEMENT

Water resources in many respects defy rational description. Water is a vital and unique resource in relation to human societies and economies and in the management and protection of the world's natural and ecological resources. Not only is it impossible to substitute another substance to fulfjl water's vital biological functions within living organisms, including human beings, but its complex multifaceted, multifunctional characteristics set it apart from any other natural resource and place it beyond the scope of most of the established approaches for planning and managing the rational utilisation of these resources. It is even debatable whether water should be regarded as an entirely "manageable" resource or whether its quantification will ever be achievable in a comprehensive and precise manner.

The formal and informal means by which water resources are controlled, both politically and economically are complex and varied. In most instances the current sytems are unsuited to resolve the conflicts of interest or priority which arise and may even exhacerbate them. they have also been criticised because they lead to inefficient use of resources and fail to protect the resource base and the environment. Although some people have argued the need for a method of evaluation and control based on monetory values as a prerequisite to efficient resource management, it is questionable whether this can be universally applied in the case of water.

In facing this complexity, it is not surprising that governments and professionals have drawn back from adressing water management in a comprehensive manner. In most cases, they have concluded that it is sufficient to specialise in a small part of the overall water economy in order to meet specific short-term needs. Planners in many of our countries have concentrated particularly on the efficient deployment of capital believing that the main constraint to efficient water management is the lack of infrastructure constrained by the lack of capital. Another way in which planners and managers have sought to achieve a simplification of the complex task they face has been simply to limit the time scale.

Water resources systems are biological, physical, sociological, economic, political, legal, geological and agricultural as well. The relative ease with which one of these aspects might be quantifiable as compared to another, does not in any way reflect a correspondingly greater importance. An approach that does not recognize and integrate these many quantitative and non quantitative dimensions of the system to the greatest extent possible can only produce an academic exercise at best. A more likely result will be a serious, perhaps irreversible, mismanagement of this vital resource. It is with reference to this complexity that the reader is invited to put the application of the optimization techniques described in the following sections into perspective.

GENERAL REQUIREMENTS AND CHARACTERISTICS

Quantitative methods for defining and evaluating alternative water resources plans include a variety of mathematical techniques. These techniques are drawn from the subject area that has been labeled systems analysis, systems engineering,operations research, or management sciences. for most purposes these terms are synonymous.

There are five major steps in using optimization models in practice:

1- Understanding the real problem;
2- Formulating the model;
3- Gathering and generating the input data for the model;
4- Solving the model;
5- Implementing the solution.

In general there is a certain amount of iteration over the five,e.g., one does not develop the most appropriate model the first time around. Of the above steps 1,3 and 5 are if not the most difficult, at least the most time consuming. Formulating good models is an art bordering on a science. It is an art because it always involves approximation of the real world. The artistic ability is to develop simple models which are nevertheless good approximation of the reality.

In the mathematical models used to describe water resource system, the design and operating variables are called decision variables for it is the best values of these variables which are to be determined. They may include, for example, the capacities of various reservoirs and pipelines of a water supply distribution system, or the allocation of land and water to various crops at an irrigation project, or the location and capacity of various flood control reservoirs and levees along a developed river.

Typical optimization models generally include at least one objective function that is either to be maximized or minimized and which serves to rank the alternative solutions or plans. In virtually every case the objective function is a scalar function; that is usage has limited the term to the determination of those quantitaive objectives which are fully commensurate. Water resources systems are particularly troublesome in this regard because there are many noncommensurate and nonquantitative objectives as previously discussed.

In addition to an objective, optimization models incorporate a number of requirements which are formulated as constraints. It is important to distinguish the different roles played by the objective function and the constraints. The optimal solution is a plan that achieves the largest (or smallest) value of the objective while satisfying all the constraints. Constraints can be of two kinds. One type of constraint expresses an actual physical limitation that cannot be violated at any cost. Such limitations may include the conservation of mass, the magnitude of fixed resources, or the capacity of existing or proposed facilities. The second type of constraint is in some sence an implicit objective or goal which in fact could be violated, although the cost of such violation may be high. Such constraints include restrictions on minimum streamflows to maintain water quality, schedules of water deliveries, and budgetary limitations. When goals are formulated as constraints, all feasible solutions must satisfy these goals. There is no explicit incentive for overfulfilling these goals, nor are the goals permitted to be reduced if the cost of meeting them is too high. These adjusments are often made after an examination of the optimal solution of the problem as initially formulated. Occasionally, deciding whether a requirement should be a constraint or an objective is difficult, simply because objectives are not well defined at the beginning of the formulation process.

There are two basic approaches for solving water resource systems problems: optimization and simulation. Optimization includes a diverse set of techniques among which Linear programming and Dynamic programming are the most commonly used. Simulation relies on trial-and error to identify near-optimal solutions. The value of the decision variable is set, and the resulting objective values are evaluated. the difficulty with the simulation approach is that there is often a frustratingly large number of feasible solutions or plans. Even when combined with efficient techniques for selecting the values of each decision variable, an enormous computational effort may lead to a solution that is still far from the best possible.

The choice of methods depends on the characteristics of the problem being considered, on the avilability of data and the on the objectives and constraints specified. Combinations of different methods have also been reported in the literature. Linear and dynamic models and simulation procedures are considered in the following sections.

LINEAR PROGRAMMING

Linear Programming (LP) has been one of the most widely used techniques in water resources management. It is concerned with solving a special type of problem: one in which all relations among the variables are linear, both in constraints and in the objective function to be optimized.

A typical LP model is

$$\text{Min } Z = C^t X$$

subject to

$$AX >= b \quad X >= 0$$

in which

C	n-dimensional vector of objective function coefficients;
X	n-dimensional vector of decision vaiables;
b	m-dimensional vector of right hand sides;
A	m x n matrix of constraint ("technological") coefficients;
t	transpose operation.

the application of LP to water resources management vary from relatively simple problems of straightforward allocation of resources to complex situations of operation and management. Under certain assumptions, nonlinear problems can be linearized and solved.

The essential advantages of LP include (1) its ability to accomodate relatively high dimensionality with comparative ease, (2) universal optima are obtained, (3)no initial policy is needed and (4) standard computer codes are readily available[2]. The irrigation planning example given in appendix I illustrates an application of linear programming. This example has been selected to demonstrate the integration orientation of optimization models and the need not only of an adequate knowledge of resources and the constraints within which the resources must be managed but also of an adequate and reliable database without which management is a difficult task.

---

2 Unlike most other optimization techniques, commercial Linear Programming packages are available even on microcomputers. Hence to use linear programming it is not necessary to understand all the details of the linear programming solution procedures. This a distinct advantage over most other types of optimization which has created the incentive to structure many nonlinear as well linear water planning problems as linear optimization models.

DYNAMIC PRORAMMING

Dynamic programming (DP), a method formulated largely by Bellman [1957], is a procedure for optimizing a multistage decision process. DP is used extensively in the optimization of water resource systems. The popularity and success of this technique can be attributed to the fact that the nonlinear and stochastic features which characterize a large number of water resources systems can be translated into DP formulation. In addition, it has the advantage of effectively decomposing highly complex problems with large number of variables into a series of subproblems which are solved recursively.

It is not unusual to find that a problem can be formulated in more than one way, and part of the art of DP lies in deciding the most efficient formulation for the problem at hand. For example, stages may represent different points in time or in space, and states may be continuous rather than discrete.

Following closely the notation of Bellman, it is assumed that the problem system may be modeled through a st of the following mathematical concepts of objective function, state variables, control or decision variables and transformations. Formally then we consider a state vector $X_t = (x_t(1), x_t(2), ... x_t(n))$ which describes the system at the start of the i^{th} stage. Each element (1,2,..,n) of the vector X_t measures a different property of the system, n being the dimension of the state vector. The policy or decision vector $Q_t = (g_t(1), g_t(2), ... g_t(r))$ is the vector of r decision variables to be selected at each stage from the set of allowable vectors $S(Q_t)$.

The state of the system at the t+1 stage is then determined by the vector of stage-to-stage transformation equations T_t whereby:

$$X_{t+1} = T_t(X_t, Q_t)$$

For an n stage decision problem we concern ourselves here with the problem of maximizing an objective function of the form:

$$R(X_1, X_2, ...X_N, Q_1, Q_2, ...Q_N) = \sum_{t=1}^{N} h_t(X_t, Q_t)$$

The problem is to select a sequence of decision vectors $Q_1, Q_2, ...Q_n$ which maximize R.

The maximization procedure results from applying the principle of optimality:"an optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision."

Let the maximum value of R, which depends only on N, the number of stages and X the initial state, equal $F_N(X)$. Then by the principle of optimality the basic functional equation is obtained:

$$F_1(X) = \max_{Q_1} h(X_1, Q_1$$

$$F_N(X_1) = \max [h(X_1, Q_1) + F_{N-1}(T(X_1, Q_1))] \quad N>=2$$

The solution is then determined by making repeated use of the above functional equations. The procedure is demonstrated in the example given in appendix II where DP is applied to the determination of reservoir releases.

SIMULATION

Simulation is perhaps the most powerful of all the tools available to water resource systems analysts, but the reason of its extraordinary power lies in its mathematical simplicity rather than its sophistication. It is a modeling technique that is used to approximate the behavior of a system on the computer, representing all the characteristics of the system largely by a mathematical description. It is different from a mathematical programming technique. Mathematical programming techniques find an optimum decision meeting all system constraints while maximizing or minimizing some objective. On the other hand, the simulation model provides the response of the system for certain inputs, which include decision rules, so that it enables decision maker to examine the consequences of various scenarios of an existing system or a new system without actually building it. A mathematical programming model usually requires assumptions on model structure and system constraints for practical implementation, whereas a simulation model is more flexible and versatile in simulating the response of the system. On the other hand, optimization looks at (implicitly) all possible decision alternatives, while simulation is limited to finite number of input decision alternatives. A typical simulation model for a water resources system is simply a model that simulates the interval-by-interval operation of the system with specified inflows at all locations during each interval, specified system characteristics and specified operating rules.

Simulation models have been successfully used by various practitionners often in conjunction with streamflow synthesis. Indeed the two techniques are close partners among the arsenal available to water-resource planners. In recent years a tendency has been toward incorporating an optimization scheme into a simulation model to perform certain degrees of optimization. It has been quite common to have a few optimization routines nested in a simulation model. Because of the complexities of water resources systems and noncommensurable objectives in water resources management, simulation is an effective tool for studying the operation of the complex water resource system incorporating the experience and judgment of the planner or design engineer into the model.

CONCLUSION

The application of systems methods such as mathematical optimization and simulation can significantly aid in the definition, evaluation and selection of water resources investments, designs, and policies. There is increasing use and documentation of successful field applications of these techniques. Although this is still a very active area of research, a large amount of knowledge and practical experience is already available as a basis for action.

It is an important task for us within CIHEAM to make these techniques known and to develop them so that they can be applied by all those concerned by the rational development and use of our water resources especially in the context of impeding water shortages and growing concerns about sustainability in the Mediterraneen region. More specifically, it is our belief that incorporating courses on these techniques in the training curricula of our institutions will contribute to improving the water consciousness and water management ability of the future decision-makers and practioners as it will make them:

(a) Better informed as to the full dimensions of water resources development, the benefits as well as the costs;

(b) More sensitive to the desirable and undesirable effects of resources development including environmental impacts;

(c) Familiar with each component of the analysis;

(d) More confident of their ability to offer judgements and make decisions.

It is hoped that CIHEAM will take the necessary actions to promote training and research in this important field which has already proved to be an indispensable aid to managerial decision making.

REFERENCES

Ait Kadi M., Recherche Operationelle, Programmation Lineaire, Notes de Cours, IAV Hassan II, 1986

Biswas A.K. et al Sustainable Water Development and Management: A Synthesis, UNEP, March 1989.

Bolton P, HR Personnal Communication

Hall and Dracup Water Resources Systems Engineering, McGraw-Hill Series in Water Resources and Environmental Engineering,McGraw-Hill

The Dublin Statement International Conference on Water and the Environment - Devlopment Issued for the 21st Century 26-31, Dublin Ireland

ICID The Application of Systems Analysis to Problems of Irrigation, Drainage and Flood Control, pergamon Press

Loucks D. P. et al Water Resource Systems Planning and Analysis, Prentice-Hall

Voturba L. et al Analysis of Water Resource Systems, Development in Water Science, 32, Elsevier.

William W-G. Yeh Reservoir Management and Operations Models: A State-of-the-Art Review, Water Resources Research, Vol 21, No 12, pp 1797-1818, Dec. 1985