Use of mathematical models in water resources and water management studies

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L'eau

Paris : CIHEAM
Options Méditerranéennes; n. 14

1972
pages 72-75

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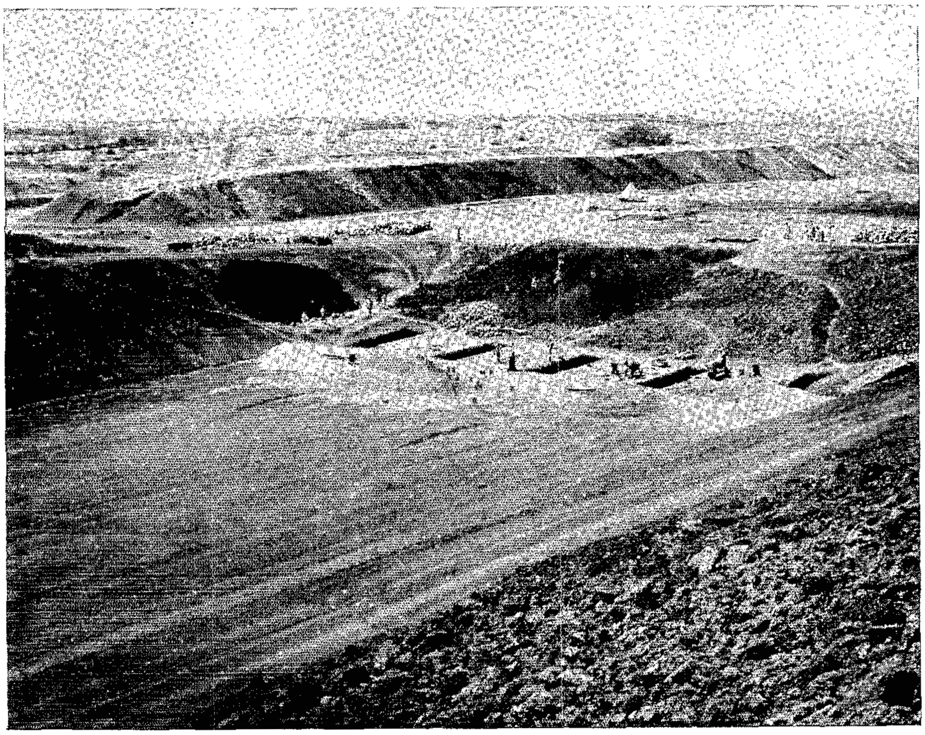
Photo Sudan Government



Construction of one of the big regulaton on the main canal of the Managil Extension (Managil Irrigation Scherne, between the Blue and White Nile Rivers).

To determine statistics, such as the expected frequency of droughts in a particular stream, the water resources planner may use either a very large sample of flows from the stream in question or the exact probability distribution of flows in the stream.

Hydrology is unable however to provide either the exact probability distribution of flows or long records of past flows.

One answer to this dilemma is the use of hydrological models.

Hydrological models exist already long time but their utility was limited due to the tremendous amount of manual calculations required. The computer has solved this problem and nowadays a considerable, still increasing number of hydrological models are available at scientific level. For the engineer in the field this considerable number causes confusion, unfortunately and he finds difficulties in applying these models. Moreover, these models often have been tested with convenient and well chosen historic records whereas the field engineer has to find a convenient model for the historic record available.

This paper therefore presents first a general outline of hydrological models and there usefulness before embarking on a discussions of the use of some hydrological models in the UNDP/FAO - LEBANON-13 Project.

Definition of hydrological models

In these chapters intensive use will be made of work done by Clarke, O'Donnell and others (See references (1) and (2)).

Some definitions need to be given :

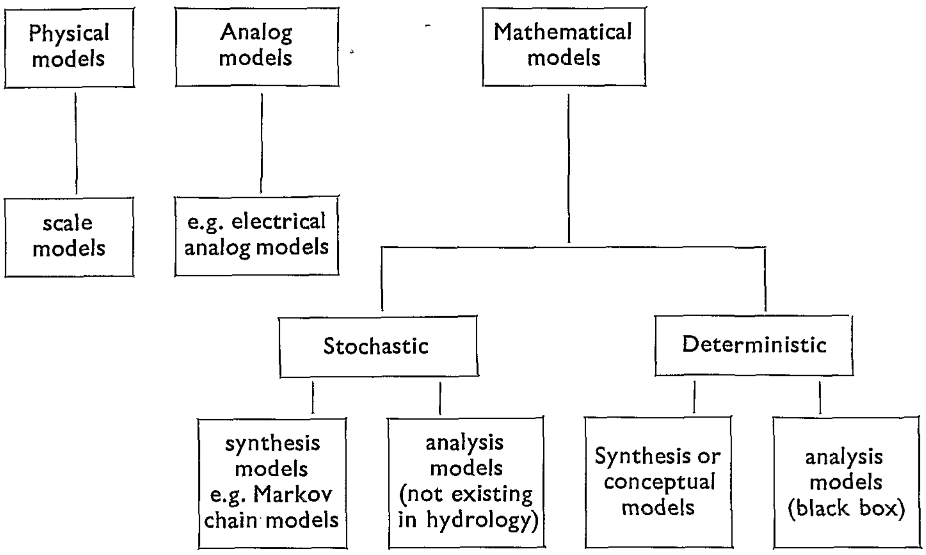
— A hydrological system is a set of physical, chemical and/or biological processes which act upon an input variable (or variables) to convert it (them) into an output variable (or variables). This variable is a characteristic of a system which may be measured and which assumes different values when measured at different times.

— A model is a simplified representation of a complex system, and hence a hydrological model is a model of a hydrological system.

Model classification

Hydrological models may be either physical, analog or mathematical. Table 1 shows a suggested classification of hydrological models.

TABLE 1. — **Hydrological models**



A physical model can be a scaled-down laboratory facsimile of the full scale system or prototype, an analog model can be, an electrical analog of a flow pattern through a dike, an aquifer, a flood hydrograph, etc. Mathematical models represent the behaviour of the system by a set of algebrais or differential equations.

This paper deals exclusively with mathematical models.

MATHEMATICAL MODELS IN HYDROLOGY

Table 1 shows two main categories of mathematical models: Deterministic and stochastic.

If all the variables of the mathematical model are free of any distribution in probability then the model is said to be *deterministic*. Examples are : the conventional routing of flood through a reservoir, the mathematical formulation of the unit —hydrograph theory and the simulation models of catchment behaviour by consideration of the physical processes acting upon the input variable (hence the concept that a hydrological variable has a probability structure is excluded). " These catchment models attempt to embody the model builder's concepts of how catchment component processes occur and are interconnected. The processes and interconnections are expressed by general parametric equations based on the known or assumed physics of catchment behaviour. Any such general model is fitted to a specific catchment by finding that set of numerical values of the model parameters such that an actual observed sequence of stream flow on the catchment can be matched by the model-computed stream flow when the model is supplied with the rainfall and potential evaporation data corresponding to the observed stream flow.

Generally, deterministic models can be used only for small to medium sized homogeneous catchments and can only operate with data intervals of the order of a day or less. For the fitting procedure, they require at least three or four years of continuous concurrent records of rainfall, potential evaporation and stream flow ". O'Donnell, Ref (1).

Stochastic models incorporate the concept of a probability structure of hydrological variables. Again quoting O'Donnell: " In essence, these models seek to identify the underlying probabilistic structure of hydrological variables (e.g. daily, monthly or annual stream flow). Information is obtained by using statistical methods on an observed chronological sequence of such a variable. Once identified and expressed in quantitive terms, such a stochastic (or probabilistic) structure can be used to generate synthetic (non-stochastic) events that may be taken as coming from the population of which the base historic record is a sample.

In contrast to deterministic models, there is no attempt to incorporate the physics of catchment behaviour in a stochastic model.

The stochastic models of streamflow can be used for all sizes of catchment whether homogeneous or not ".

These models can treat either only one variable, often stream flow, or combinations with more variables (e.g. temperature and rainfall). No firm rule can be laid down about the minimum length of observed record needed, but ten years could be considered to be a minimum (if no valuable correlation exists with another longer record in which case the minimum length could be shorter).

OBJECTIVES OF MODEL STUDIES

Before embarking on any modelling study it is vital that the objectives are clearly defined.

Broadly speaking, there are two main uses of hydrological models : *forecasting* and *prediction*. Forecasting tries to assess what is going to happen in the immediate future whereas prediction is for design purposes e.g. reservoir design or the determination of the expected frequency of droughts (of a given severity) in a particular stream.

CHOICE OF MATHEMATICAL MODEL

The diversity of problems found in engineering studies and the types of information available (long records, short or very short records, and even virtual absence of any observations at all) has important implications for the choice of method for solving the problems.

As Bernier (1) states it: " It is important that the interrelationship between three basic elements should be fully appreciated. These three elements are:

a) available hydrometeorological information, and its limitation;

b) mathematical models (deterministic or stochastic) which can be used to mobilize the information;

c) economic problems, whose solutions will depend, in their precision, on both the information available and the models.

The development and application of a particular model (deterministic or stochastic) brings together on the one hand quantitive (" objective ") information obtained in the field and on the other hand the preconceived (" subjective ") ideas of the hydrologist on the basin under study. As the amount of objective information decreases, so the importance of subjective knowledge increases. The particular model adopted in any specific case therefore depends on the balance between these two types of information.

Concerning the choice between deterministic and stochastic models the following remarks can still be made :

— The great number of parameters in deterministic models that must be adjusted against the observations puts these models in a weak position compared with stochastic models. However, they do have the advantage that they can incorporate the " a priori " catchment area knowledge of the hydrologist.

— Deterministic modelling has limited use for design purposes and is primarely an operational tool. The latter becomes particularly interesting when long generated stream flow records are used as input data e.g. in surface —and ground-water— reservoir analysis studies.

— Stochastic models are aimed at prediction purposes with particular importance for design and flood magnitude/frequency studies.

GAINS TO BE OBTAINED BY THE USE OF HYDROLOGICAL MODELS

Existing stream flow records are not sufficiently extensive to provide estimates of many important statistics, e.g. the proportion of years with droughts of a given severity. One answer to this dilemma is to generate records which are close enough to possible (but not observed) historical records that they may be used to estimate these statistics. These generated or synthetic stream flows are therefore extremely useful planning tools. The length of such a record depends on what is needed for any given design purposes but can be many times the historic record and certainly several times the expected project life. The long generated record can then be routed through any trial reservoir to assess its performance.

The gain to be obtained by deterministic modelling can be: better forecasting and knowledge about the influence of land use changes. The latter includes also reconstruction of missing stream flow records provided that long records of rainfall and potential evaporation exist. Another use of a deterministic model of the Mero type (see description of the Lebanon Project hereafter) has made possible the simulation of the accumulation and melting of snow in order to study the influence of snow melt on summer flows.

APPLICATION OF MATHEMATICAL MODELS TO A PROJECT IN NORTH LEBANON

Background

One of the objectives of the UNDP/FAO Project LEB 13, concerning the Hydro agricultural development of North Lebanon, was to study an irrigation scheme of about 7 000 ha in the Koura Zghorta region. For the water supply of this scheme a dam has to be constructed on the Aasfour river. The reservoir inflow will be provided by the Aasfour discharges together with part of the stream flow of an adjacent river.

For the routing of these stream flows through the reservoir only short historic records (ranging from 3 to 12 years) were available. The presence of outliers (very wet and several consecutive dry years, made it virtually impossible to establish with any confidence a return period for these outliers.

The application of the conventional technique of correlating the short stream flow record with long rainfall records was only possible on *annual* basis. Very low values of the correlation coefficient between *monthly* rainfall and stream flow were found due to the influence of snow and/or springs on the catchment response to rainfall. Unfortunately monthly stream flow data are needed for reservoir analysis studies.

As the conventional techniques were unable to provide these data, the use of hydrological modelling techniques became necessary.

Choice of the model (or models) to be applied
(see also table 2)

The UNDP/FAO Project in Cyprus had used a deterministic catchment model, known under the name of Mero model, which, can be considered as a simplified Stanford model. This model was tested on some Lebanese catchments and the resulting simulated stream flows were rather satisfactory, especially after a snowmelt routine was added.

The length of these reconstructed stream flow records however never exceeded 30 years, these 30 years being the longest available rainfall record in the area under study. It was felt that these 30 years of stream flow record were not enough for reservoir analysis studies of a dam with a project time of 50 years chosen for the economic calculations.

It was then that the Project Consultant, Mr. J. Bernier, Chief of the Statistics Group at the Laboratoire National d'Hydraulique de Chatou (France), suggested the use of stochastic modelling techniques and proposed a number of such models. Consequently five generating models were constructed and long series of stream flow, rainfall and temperature could be generated.

The generation of long series of rainfall and temperature became necessary in order to calculate long series of water demand. To avoid generating series of stream flow, rainfall and temperature that were uncorrelated, a method of " in phase " generation was adopted ensuring for example that a dry year would be caracterised by low values of stream flow and rainfall, together with high temperature values resulting in high demand for the year.

Methodology for adjusting water resources and water demand as applied to north Lebanon

The availability of generating models, as described above, permitted to apply a methodology of which the principle stages are:

I. — Analysis of existing data and choice of method of generation.

II. — Generation of long series of rainfall, temperature and stream flow mutually in phase. Calculations were on a monthly basis except for the rainfall — stream flow correlation which was only possible on an annual basis.

III. — Calculation of long series of water demand, first by crop and then for the

TABLE 2. — **Models applied in Lebanon in water development studies**

Objective	Type of model	Input data (monthly values)	Output data (monthly values)
Water resources evaluation	Stochastic models	Historic record of rainfall and stream flow	Long generated series of rainfall and stream flow
	deterministic model (Mero)	d°	Simulated series of stream flow with a length equal to the historic rainfall record.
Water demands estimate	deterministic model (Blaney-Griddle)	Long series of generated rainfall and temperature	Long generated series of water demand, per crop and for total irrigable area
Adjustment of water resources and water demands	deterministic surface water reservoir model	long series of stream flow and water demand	Irrigation scheme operation — reservoir size — irrigable area — shortages (time series) — effects on other water users
	deterministic groundwater reservoir model	d°	d°

irrigable area, using the Blaney Griddle formula.

IV. — Adjustment of the series of generated stream flows to take into account the constraints on the usability for the new scheme of these flows, for example: present water use, maximum capacity of a diversion tunnel, priorities in water use (hydropower versus irrigation, etc.).

V. — Simulation of cheme operation, giving for each scheme the reservoir capacity, irrigable area, shortages, etc., using *one* generated series of 50 years.

VI. — Systematic economic comparison of all the solutions found under V (calculation of internal rate of return) and choice of the optimal solution (irrigable area and reservoir under given management hypothesis concerning the degree of failure acceptable, priorities of water use, etc.).

VII. — Final and more detailed simulation of the chosen system using 20 series of 50 year generated data on rainfall and temperature (for calculation of water demand) and stream flow.

The choice of methodology for North Lebanon (for example use of Blaney-Griddle formula and first priority being given to stochastic methods) was in response to the availability of data and other local conditions such as:

a) the presence of two records of observed stream flow of 12 years length ;

b) the absence of a good correlation between monthly rainfall and monthly stream flow (due to snowfall and karsticity);

c) a good correlation between rainfall and stream flow on an annual basis;

d) the perennial flow of the rivers.

The case of a discontinuous flow series (summer flows zero) has not therefore been considered in these studies but no doubt it could be treated by appropriate statistical techniques.

In a simular way if no " long " series (at least 10 years) had been available another approach would have been necessary.

In the case of an almost total lack of stream flow data (for example two years of record only) a better approach could be to apply a deterministic model (assuming that daily rainfall records are available).

Computer programmes

All the programmes for the calculation have been written in FORTRAN IV for the IBM 1130 (16 K) computer. These programmes have been written in such a way that they can be used separately or in series. In this latter case the calculation time is one hour per run (stages II through V, see above). To obtain as output in one run the results of system simulation (stage V) the following input data are needed:

a) a historic record of rainfall (monthly);

b) a historic record of mean temperature (monthly);

c) a " long " (12 to 14 years in North Lebanon) historic record of stream flow (monthly);

d) a " short " (3 to 5 years) historic record of stream flow (monthly);

e)* the duration of sunshine as a percentage of the maximum possible;

f)* the monthly crop coefficient K (by crop);

g) the maximum usable soil moisture storage (by crop);

h) the coefficient of growth of the plant (by crop);

i) the phasing of irrigation development of the whole area, and the subdivision of the area by crop (in %).

j) the rate of change-over from the present olives groves to the new crops;

k) coefficient of irrigation efficiency;

l) geometric characteristics of the reservoir.

Gains obtained by this methodology

The use of stochastic models, resulting in long generated series of stream flow, rainfall and temperature, has permitted:

1. a much better estimate of the statistics, such as the expected frequency of droughts of a given severity. This alone could justify the use of such models.

2. a better understanding and estimate of the interannual variability of crop water demands. The generation " in phase " in flows, rainfall and temperatures, leading to " in phase " supplies and demands from a design reservoir, is a marked improvement over the more usual treatment of assuming a fixed annual cycle of demands.

3. to assess in a more refined way the performance of the design reservoir resulting in a better estimate of the optimum size.

4. the economists to take into account for their economic evolution the expected frequencies of droughts of given severities (e.g. lower crop yields in dry years). It was in this way also possible to evaluate the influence of the place of droughts in a 50 years series (actualisation).

5. the testing of a considerable number of hypothesis. Because operational hydrology provides large numbers of flows for use in evaluating storages, electronic computers are needed.

Stochastic input in other models

The methods described above are using the generated flow records as input data for the (deterministic) reservoir simulation model. The same procedure however can be used to assess the performance of groundwater storages. This has been applied by the LEB 13 Project to a deterministic model of a groundwater aquifer in the northern Akkar plain in Lebanon (3). This way of simulation permitted a much better understanding of the performance of this aquifer especially during consecutive dry years.

(*) *e* and *f* are necessary for the application of the Blaney-Griddle formula.

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Hydrological models are extremely useful and powerful planning tools and their use improves the water resources planning process significantly.

Although several successful applications of hydrological models to field problems have proven that a number of them can be said to be operational, it still is true that more effort should be given to fill the gap between science and application in order to make better use of recent developments in practical engineering problems.

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